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Discrete Optimization

Frequency optimization in public transportation systems: Formulation and metaheuristic approach

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ABSTRACT

We study the transit frequency optimization problem, which aims to determine the time interval between subsequent buses for a set of public transportation lines given by their itineraries, i.e., sequences of stops and street sections. The solution should satisfy a given origin–destination demand and a constraint on the available fleet of buses. We propose a new mixed integer linear programming (MILP) formulation for an already existing model, originally formulated as a nonlinear bilevel one. The proposed formulation is able to solve to optimality real small-sized instances of the problem using MILP techniques. For solving larger instances we propose a metaheuristic which accuracy is estimated by comparing against exact results (when possible). Both exact and approximated approaches are tested by using existing cases, including a real one related to a small-city which public transportation system comprises 13 lines. The magnitude of the improvement of that system obtained by applying the proposed methodologies, is comparable with the improvements reported in the literature, related to other real systems. Also, we investigate the applicability of the metaheuristic to a larger-sized real case, comprising more than 130 lines.

1. Introduction

When designing a public transportation system, the planners take decisions that impact in the cost of the system, which is determined by the monetary cost of operation and fares, and the travel time of the users. In systems based on buses, the literature identifies five stages for designing a public transportation system (Ceder & Wilson, 1986): route network design, frequency setting, timetable design, fleet assignment and crew assignment. In real systems, usually these stages are performed sequentially, where decisions taken at a given stage influence decisions taken at subsequent stages. Also, these decisions are taken for different planning horizons, whether the context of the planning is strategic (long term), tactical (medium term) or operational (short term).

The frequency setting problem implies to determine the time interval between subsequent buses on the lines, based on their itinerary (sequence of street segments and bus stops) and the demand given by an origin–destination matrix. During the strategic planning of a public transportation system (in particular, when designing the itinerary of the lines, i.e. the route network), a preliminary setting of frequencies is needed. Also, during the tactical planning it is necessary to adjust the frequencies to demand variations along different seasons of the year or time of day, or as response to changes in the route network design (Desaulniers & Hickman, 2007). The frequencies impact both on the users (waiting time, capacity of the lines) and also in the operators (operational cost determined strongly by the size of the required fleet).

The frequency setting problem has been approached in the literature as an optimization problem, where usually the objective function states the minimization of the overall travel time of the users (walking, on-board and waiting), under a fleet size constraint as well as other infrastructure and policy constraints (Constantin & Florian, 1995; Han & Wilson, 1982; Schéele, 1981). Since frequency optimization models should include measures relative to the performance of the systems from the viewpoint of the users (typically the waiting time), they should include a sub-model of the behavior of the users with respect to a set of bus lines. Such a model, known as *assignment sub-model*, usually has a complex formulation and solution method, specially when the influence of the bus capacity is considered in the modeling of the user behavior. That complexity determines an important part of the overall complexity of the frequency optimization model. Moreover, the validity of an assignment model for public transportation, in most cases depends on the real context where it is applied.

The existing studies concerning frequency optimization usually involve nonlinear models which are solved approximately (Constantin & Florian, 1995; Schéele, 1981). The nonlinearity arises from the fact that the waiting time is inversely proportional to the frequencies; also, the modeling of the interaction among different

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lines results in nonlinear expressions. The existing models consider assignment sub-models exhibiting different degrees of realism. The cases used to test the methodologies range from small-sized and fictitious cases to medium-sized cases related to real cities comprising up to 100 lines approximately (Constantin & Florian, 1995; Yu, Yang, & Yao, 2010).

Taking into account this state of the art (more widely discussed in Section 2), in this work we contribute in two specific directions:

- We propose a mixed integer linear programming (MILP) formulation for an already existing frequency optimization model (Constantin & Florian, 1995). Given the nature of the proposed formulation, it can be solved exactly by using a commercial MILP solver. Moreover, it includes an assignment sub-model (Spiess & Florian, 1989) that is widely accepted in the literature.
- With the aim of solving large-sized instances of the problem (systems comprising more than 100 lines), we propose a metaheuristic which accuracy (in the sense of distance to optimum) is estimated by comparing against results produced by the exact model (when possible, i.e. for the smaller instances).

Concerning the first contribution, we note that no method existing in the literature is able to find solutions with proven (global) optimality. This is particularly important in the transit frequency optimization problem, since the improvements reported over user's travel time of current solutions are relatively small (Constantin & Florian, 1995; Yu et al., 2010). Therefore, given the heuristic nature of existing solution methods, it remains unclear whether is possible to improve the results even more.

Our metaheuristic is able to find solutions which accuracy is estimated (when possible) by comparing against exact solutions. To the best of our knowledge, this is the first published approximated method which results are validated against optimal ones using a real case. The metaheuristic produces results in relatively short time and it includes routines which execute in polynomial time (with respect to the number of lines, size of the underlying graph model and density of the origin–destination matrix), so it is suitable to be applied for solving larger cases.

The models and algorithms proposed in this work are applied to real cases and fictitious ones, which illustrate their applications.

The structure of the article is as follows. Section 2 presents a literature review while Section 3 contains a description of the mathematical model and the proposed mathematical formulation. Section 4 describes the metaheuristic proposed to solve the problem approximately while Section 5 presents numerical results of both exact and approximated methods over different test cases. Finally, Section 6 comments conclusions and further work.

2. Literature review

In this section we provide a review of representative studies in the field of frequency optimization for public transportation systems. Almost all models are formulated in terms of a graph which nodes represent bus stops, centroids (fictitious points where the demand of a given zone is assumed to be concentrated) or endpoints of a section of line's itinerary. The arcs represent either a section of a line's itinerary, a walking trajectory (between centroids and stops) or a specific event or action, like waiting for a line or performing a transfer between different lines. Moreover, typically it is assumed that the demand between different zones of the city (represented by centroids) is given in the form of an origin–destination (OD) matrix; each element different from zero of this matrix is called *OD pair*. It is worth mentioning that different levels of detail of the graph model mentioned above can be found in the literature.

The model proposed in (Schéele, 1981) states the minimization of the walking and on-board travel time plus the waiting time. A constraint imposes an upper limit on the fleet size. The behavior of the users is implicitly embedded into the model: given an OD pair, its demand is divided among the different lines according to an entropy and a bus capacity constraint. The formulation has a nonconvex objective function and linear or convex constraints. The solution algorithm computes an approximated solution by refining a set of frequencies according to a descent strategy. The methodology is tested with a case relative to the city of Linköping (Sweden), with 6 lines and 38 zones.

In (Han & Wilson, 1982) a model is proposed to set frequencies on heavily utilized lines. Therefore the objective function states the minimization of the occupancy level at the most heavily loaded point on any route in the system. The constraint set includes upper limits on the fleet size and the capacity of the buses. The assignment sub-model is represented by a nonexplicit constraint which encodes the hypothesis concerning the user behavior: passengers give preference to lines that lead to destination directly (i.e. without transfers), although they imply higher travel time. Besides this rule, the demand corresponding to a given OD pair is distributed among the different lines following the frequency-share rule (Chriqui & Robillard, 1975). A two stage heuristic is proposed to solve the model: first, a base allocation procedure (which iteratively corrects passenger flows and line frequencies) is performed in order to find a lower bound for the bus capacity constraint; second, a surplus allocation procedure solves a problem with only linear constraints. Although the methodology was proposed to be applied to the city of Cairo (Egypt), only an illustrative case comprising 6 nodes and 3 routes is shown.

The frequency optimization problem is stated in (Constantin & Florian, 1995) as a nonlinear bilevel problem. In this model, the upper level represents the planner who wants to ensure minimal overall travel time and fleet size feasibility. The lower level represents the users who act by minimizing the travel time, according to the optimal strategies assignment model (Spiess & Florian, 1989). Therefore, the objective functions of both levels have the same expression. The model is solved approximately by an iterative algorithm based on a gradient descent which uses specific properties of the problem. The methodology is tested by using cases related to the cities of Stockholm (Sweden), Winnipeg (Canada) and Portland (U.S.A.), comprising 38, 67 and 115 lines respectively.

In (Gao, Sun, & Shan, 2004) a multi-objective model is proposed, which seeks to minimize the overall travel time of the users and the operational cost of the operators (assumed to be linearly proportional to the frequencies). The salient characteristic of this work is the internalization of the congestion in the behavior of the users. For a given set of frequencies, the assignment model proposed in (de Cea & Fernández, 1993) is applied, which distributes the demand according to the effective frequencies. The proposed approximate solution method starts with an initial set of frequencies, which is successively improved by a sensitivity analysis procedure. The methodology is tested by using a very small illustrative example comprising 4 nodes and 4 lines.

More recently, Yu et al. (2010) propose a genetic algorithm for bus frequency optimization. To the best of our knowledge, this is the first application of metaheuristics to this problem. The optimization model considers the minimization of the on-board and waiting time, subject to a fleet size constraint. The behavior of the users is modeled by using the optimal strategies assignment model (Spiess & Florian, 1989). The approximate solution method uses an integer encoding of frequencies and genetic operators which redistribute the available fleet among the different lines of the system. The methodologies are tested with an illustrative small-sized case and with a case related to the city of Dalian (China) comprising 3,004 bus stops and 89 lines.