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Discrete Optimization

Complexity results for the linear time–cost tradeoff problem with multiple milestones and completely ordered jobs ☆

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ABSTRACT

We consider two linear project time–cost tradeoff problems with multiple milestones. Unless a milestone is completed on time, penalty costs for tardiness may be imposed. However, these penalty costs can be avoided by compressing the processing times of certain jobs that require additional resources or costs. Our model describes these penalty costs as the total weighted number of tardy milestone. The first problem tries to minimize the total weighted number of tardy milestones within the budget for total compression costs, while the second problem tries to minimize the total weighted number of tardy milestones plus total compression costs. We develop a linear programming formulation for the case with a fixed number of milestones. For the case with an arbitrary number of milestones, we show that under completely ordered jobs, the first problem is NP-hard in the ordinary sense while the second problem is polynomially solvable.

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1. Introduction

The time–cost tradeoff problem (TCTP) assumes that processing times can be compressed through the expenditure of additional resources such as labor and capital (Ford & Fulkerson, 1962; Kelley, 1961). Its typical objective is to minimize the project completion time subject to a constraint on total costs or to minimize total costs subject to a constraint on project completion time.

The *linear* TCTP (LTCTP) is defined as a TCTP where the set of possible processing times is a closed interval and compression cost decreases linearly on the closed interval. The *discrete* TCTP (DTCTP) is defined as a TCTP with the discrete set of possible processing times. The LTCTP can be formulated as a linear programming (LP) problem. Furthermore, it can be solved efficiently with a network flow approach (Fulkerson, 1961; Kelley, 1961). However, the DTCTP is strongly NP-hard (De, Dunne, Ghosh, & Wells, 1997). Thus, for the DTCTP, Skutella (1998) developed approximation algorithms with a constant performance guarantee for various special cases. For a comprehensive review of the more general model (e.g., (multi-mode) resource-constrained project scheduling), see (Artigues, Demassey, & Neron, 2008; Brucker, Drexl, Möhring, Neumann, & Pesch, 1999; Demeulemeester & Herroelen, 2002; Weglarz, 1999; Weglarz, Jozefowska, Mika, & Waligora, 2011).

The TCTP research above assumes one milestone for the overall project, that is, the last job. In reality, however, milestones can exist at any point in the project. For example, a venture capital company makes small investments in a project at first and then, when a milestone has been reached, determines whether to stop the project or make more investments (Bell, 2000; Sahlman, 1994). To the best of our knowledge, no study has been conducted on an LTCTP with more than one milestone.

This paper considers two LTCTPs with multiple milestones such that penalty costs occur unless each milestone is reached no later than its due date. The first problem tries to minimize the total weighted number of tardy milestones within the budget for total compression costs, while the second problem tries to minimize the total weighted number of tardy milestones plus total compression costs. Let the first and second problems be referred to as LTCTP 1 and LTCTP 2, respectively.

Our problems can be formally stated as follows. The LTCTP is represented by a directed *activity-on-node* graph $G = (V, A)$, where $V = \{1, 2, \ldots, n\}$ is the set of jobs and A is the set of precedence relations. Relation $(i, j) \in A$ means that job i should be completed before job j is started. Associated with job j is a normal processing time p_j, a maximal compression amount u_j, and a compression cost rate c_j, $j = 1, 2, \ldots, n$. Let $x = (x_1, x_2, \ldots, x_n)$ be the vector, where x_j is the compressed amount of job j and $0 \leqslant x_j \leqslant u_j, j = 1, 2, \ldots, n$. Note that throughout the paper, without loss of generality, we will consider only schedules such that each job is processed as soon as possible by not allowing unnecessary idle time. Thus, since each job has a unique starting time under each x, let x be termed

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schedule in this paper. Let $D \subseteq V$ be the set of milestones. Note that each milestone corresponds to a specific job. For $j \in D$, let w_j and d_j be the penalty cost for tardiness and the due date of milestone j, respectively. Let $C_k(x)$ be the completion time of job k under x. Then, throughout this paper, LTCTP 1 and LTCTP 2 are defined, respectively, as

$$\begin{aligned} \text{minimize} \quad & \sum_{j \in T(x)} w_j \\ \text{subject to} \quad & C_i(x) + p_j - x_j \leqslant C_j(x), \quad \forall (i,j) \in A, \\ & \sum_{j=1}^{n} c_j x_j \leqslant B \\ & 0 \leqslant x_j \leqslant u_j, \quad j = 1, 2, \ldots, n, \end{aligned}$$

and

$$\begin{aligned} \text{minimize} \quad & \sum_{j \in T(x)} w_j + \sum_{j=1}^{n} c_j x_j \\ \text{subject to} \quad & C_i(x) + p_j - x_j \leqslant C_j(x), \quad \forall (i,j) \in A, \\ & 0 \leqslant x_j \leqslant u_j, \quad j = 1, 2, \ldots, n, \end{aligned}$$

where $T(x) = \{j | C_j(x) > d_j \text{for} j \in D\}$ is the set of tardy milestones in D under x and B is the budget for total compression costs. Let milestone g be referred to as a just-in-time (JIT) job under x if completed exactly on its due date under x, that is, $d_g = C_g(x)$.

Proposition 1. *LTCTP 1 and LTCTP 2 with a fixed number of milestones are polynomially solvable.*

Proof. Suppose that $\overline{D}$ is the set of milestones which are completed no later than their due dates in a feasible schedule. For given $\overline{D}$, LTCTP 1 and LTCTP 2 can be formulated as an LP below.

$$\begin{aligned} \text{minimize} \quad & \sum_{j \in D \setminus \overline{D}} w_j \\ \text{subject to} \quad & C_i(x) + p_j - x_j \leqslant C_j(x), \quad \forall (i,j) \in A, \\ & C_j(x) \leqslant d_j, \quad \forall j \in \overline{D} \\ & \sum_{j=1}^{n} c_j x_j \leqslant B \\ & 0 \leqslant x_j \leqslant u_j, \quad j = 1, 2, \ldots, n, \end{aligned}$$

and

$$\begin{aligned} \text{minimize} \quad & \sum_{j \in D \setminus \overline{D}} w_j + \sum_{j=1}^{n} c_j x_j \\ \text{subject to} \quad & C_i(x) + p_j - x_j \leqslant C_j(x), \quad \forall (i,j) \in A, \\ & C_j(x) \leqslant d_j, \quad \forall j \in \overline{D} \\ & 0 \leqslant x_j \leqslant u_j, \quad j = 1, 2, \ldots, n \end{aligned}$$

Note that since the objective value in LTCTP 1 is a constant, our concern is to find a feasible schedule. It is observed that if the set of the milestones completed before the due dates is known in advance, then LTCTP 1 and LTCTP 2 can be formulated as an LP. Since the number of the milestones is fixed, this implies that optimal schedules of the two problems can be found by solving $O(2^{|D|})$ LP's, where $|D|$ is the cardinality of D. The proof is complete. □

By Proposition 1, hereafter, we consider the case with an arbitrary number of milestones. Furthermore, in this paper, we focus on the LTCTP with a chain precedence graph (LTCTP-Chain) as starting point the LTCTP with multiple milestones. The motivation of LTCTP-Chain is found in a product development process consisting of sequential stages (Roemer & Ahmadi, 2004). Throughout the paper, assume that a chain precedence graph is described below:

$$1 \to 2 \to \cdots \to n,$$

where $i \to j$ represents that job j can start after job i is completed. Without loss of generality, assume that 1 and n are the start and the end jobs, respectively. Let LTCTP 1 and LTCTP 2 with a chain precedence graph be referred to as LTCTP-Chain 1 and LTCTP-Chain 2, respectively.

The remainder of the paper is organized as follows. Section 2 shows that LTCTP-Chain 1 with an arbitrary number of milestones is NP-hard. It also presents a pseudo-polynomial time approach for LTCTP-Chain 1 with an arbitrary number of milestones. Section 3 shows that LTCTP-Chain 2 with an arbitrary number of milestones is polynomially solvable. The final section presents concluding remarks and discusses future work.

2. LTCTP 1 with an arbitrary number of milestones under completely ordered jobs

In this section, we establish the computational complexity of LTCTP-Chain 1, and develop its pseudo-polynomial time approach.

We can transform LTCTP-Chain 1 with $D \subset V$ into the LTCTP-Chain 1 with $D = V$ by letting $d_i = \sum_{j=1}^{n} p_j$ for $i \in V \setminus D$. It is clear that optimal schedules of this two problems are identical, since jobs i for $i \in V \setminus D$ will not be completed tardily. Thus, for simplicity of notations, we assume that $D = V$.

2.1. Computational complexity of LTCTP-Chain 1

In this section, we show that the decision version of LTCTP-Chain 1 is NP-complete by the reduction from the partition problem, which is known to be NP-complete, see (Garey & Johnson, 1979). The partition problem can be stated as follows: Given n positive integers $a_1, a_2, \ldots, a_n$ such that $\sum_{j=1}^{n} a_j = A$, is there a subset $I \subset \{1, 2, \ldots, n\}$ such that $\sum_{j \in I} a_j = \frac{A}{2}$?

Theorem 1. *The decision version of LTCTP-Chain 1 is NP-complete.*

Proof. The decision version of LTCTP-Chain 1 is denoted as follows: Given a threshold K, is there a schedule x such that x satisfies the precedence relations and

$$\sum_{j \in T(x)} w_j \leqslant K \text{ and } \sum_{j=1}^{n} c_j x_j \leqslant B?$$

It is clear that decision version of LTCTP-Chain 1 is in NP. Henceforth, we will reduce the partition problem to the decision version of LTCTP-Chain 1. Given an instance of the partition problem, we construct an instance of LTCTP-Chain 1 as follows. There are n jobs such that for $k = 1, 2, \ldots, n$, let $p_k = M^k, w_k = 2a_k, c_k = \frac{a_k}{M^k}$ and $u_k = M^k$, where $M > 2(n+1)A$. Let $D = \{1, 2, \ldots, n\}$, which implies that all jobs have the dates of appraisal. Let $d_1 = 0$ and $d_k = \sum_{j=1}^{k-1} M^j, k = 2, 3, \ldots, n$. Let $K = A$ and $B = \frac{A}{2}$.

Suppose that there exists a set $\bar{I}$ such that $\sum_{i \in \bar{I}} a_j = \sum_{i \in \{1,2,\ldots,n\} \setminus \bar{I}} a_j = \frac{A}{2}$. We can construct a schedule $\bar{x}$ by letting $\bar{x}_j = M^j$ if $j \in \bar{I}$ while $\bar{x}_j = 0$, otherwise. Note that since $\bar{x}_k = M^k$ for $k \in \bar{I}$,

$$C_k(\bar{x}) = \sum_{j=1}^{k} (p_j - \bar{x}_j) \leqslant \sum_{j=1}^{k-1} M^j = d_k,$$

and thus jobs in $\bar{I}$ are non-tardy, while since $\bar{x}_k = 0$ for $k \in \{1, 2, \ldots, n\} \setminus \bar{I}$,

$$C_k(\bar{x}) = \sum_{j=1}^{k} (p_j - \bar{x}_j) \geqslant M^k > \sum_{j=1}^{k-1} M^j = d_k,$$

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