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Discrete Optimization

Genetic-algorithm-based simulation optimization considering a single stochastic constraint

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ABSTRACT

In this paper, we consider the discrete optimization via simulation problem with a single stochastic constraint. We present two genetic-algorithm-based algorithms that adopt different sampling rules and searching mechanisms, and thus deliver different statistical guarantees. The first algorithm offers global convergence as the simulation effort goes to infinity. However, the algorithm's finite-time efficiency may be sacrificed to maintain this theoretically appealing property. We therefore propose the second heuristic algorithm that can take advantage of the desirable mechanics of genetic algorithm, and might be better able to find near-optimal solutions in a reasonable amount of time. Empirical studies are performed to compare the efficiency of the proposed algorithms with other existing ones.

1. Introduction

Optimization via simulation (OvS) is the process of optimizing the expected performance of a discrete event, stochastic system through computer simulation (e.g., Abo-Hamad & Arisha, 2013; Arreola-Risa, Giménez-García, & Martínez-Parra, 2011; Chen, 2011; Hong & Nelson, 2009; Tsai, 2013; Tsai & Chu, 2012; Yu, Tsai, & Huang, 2010). Hong and Nelson (2009) classified OvS problems into three categories based on the feasible region structure: continuous OvS, discrete OvS (DOvS), and ranking and selection (R&S). For the R&S problems, the number of alternatives in the feasible region is so small (often less than 500) that we may simulate all solutions and choose the best (or near the best) among them with a specified confidence level (see Kim & Nelson (2006) for a survey). For DOvS problems, we have a very large number of feasible solutions (discrete design variables), and the existing algorithms often emphasize a global convergence to the optimal solution asymptotically. In practice, decision makers usually need to consider multiple performance measures rather than a single one due to physical or managerial requirements. For instance, in a typical flow-line problem, the decision maker is interested in finding a buffer allocation setting to maximize the expected throughput over a fixed planning horizon, while also keeping the expected overall work-in-process no greater than a certain level. Recently, more research interest in the OvS literature has been directed to solving problems with stochastic constraints or multiple performance measures. Morrice and Butler (2006) developed a R&S procedure based on multi-attribute utility theory to allow tradeoffs between conflicting targets. Kabirian and Ólafsson (2009) proposed a heuristic iterative algorithm for finding the best solution in the presence of multiple stochastic constraints. Kleijnen, van Beers, and van Nieuwenhuyse (2010) combined methodologies from metamodeling and mathematical programming for solving constrained optimization of random simulation models. Bhatnagar, Hemachandra, and Mishra (2011) and Szechtman and Yücesan (2008) proposed stochastic approximation algorithms for constrained optimization via simulation. Luo and Lim (2011) proposed a new approach that converts a constrained optimization problem into an unconstrained one by using the Lagrangian function. Similarly, Park and Kim (2011) presented a method called penalty function with memory, which is added to the objective function and then replaces a DOvS problem with stochastic constraints into a series of new unconstrained problems. Vieira Junior, Kienitz, and Belderrain (2011) proposed a novel simulation allocation rule to be used in a locally convergent random search algorithm, called COMPASS (Hong & Nelson, 2006), to handle stochastic constrained problems. Hunter and Pasupathy (2013) and Pujowidianto, Hunter, Pasupathy, Lee, and Chen (2012) applied a large-deviations approach to provide an asymptotically optimal sample allocation that maximizes the rate at which the probability of false selection tends to zero. These methods have a requirement that all the solutions be simulated at least once, so they are more appropriate to the setting where the solution space is finite and contains a small number of elements. Andradóttir and Kim (2010) presented one type of R&S procedure (called the Feasibility Determination Procedure, or FDP) that checks the feasibility of each solution among a finite

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set with respect to a stochastic constraint. Instead of giving a guarantee of correctly choosing the best solution, Bayesian procedures maximize the posterior probability of correct selection within a given simulation budget. There are some works which developed efficient Bayesian procedures to address the constrained optimization problem (e.g., Guan, Song, Ho, & Zhao, 2006; Jia, 2009; Lee, Pujowidianto, Li, Chen, & Yap, 2012; Li, Lee, & Ho, 2002). It should be noted that the existing algorithms handling stochastic constraints are more appropriate to be used when the number of solution designs is not too large. This implies that they will become inefficient (i.e., require excessive sampling cost) when applied to a very large solution space.

Most commercial OvS solvers use optimization metaheuristics, such as tabu search, neural nets, and genetic algorithms (GAs), that have generally been designed and proven to be effective on difficult and deterministic optimization problems. While these algorithms often find promising solutions quickly, they may also become pure random search methods if the stochastic variation of output (i.e., simulation noise) is high or the number of obtained samples for each solution is set too low. That is, their implementations do not always adequately account for the presence of statistical errors. In addition, these algorithms do not provide any statistical guarantee regarding the *quality* or *goodness* of the final selected solution. To handle the aforementioned issues, Boesel, Nelson, and Kim (2003) proposed an adaptive genetic-algorithm-based procedure to account for simulation noise in the stochastic optimization context. Their procedure also guarantees to return the best solution over the solutions visited by a heuristic search procedure. Subsequently, Xu, Nelson, and Hong (2010) used the niching GA together with COMPASS (Hong & Nelson, 2006) to establish local optimality with statistical confidence when simulating only a small portion of the feasible solutions. Nazzal, Mollaghasemi, Hedlund, and Bozorgi (2012) proposed a simulation optimization methodology that combines the GA and a R&S procedure under common random number (CRN). They developed a new R&S procedure to select a nonempty subset so that the best solution is contained in the subset with a pre-specified probability. Fitness values and selective probabilities are then computed based on the estimated performances which are obtained from the aforementioned R&S procedure. Notice that these genetic-algorithm-based procedures are adapted for solving the DOvS problem, but can only optimize the expected value of a single performance measure (see Ólafsson (2006) for a review of metaheuristics for OvS). To handle the multi-objective simulation optimization problem, Lee, Chew, Teng, and Chen (2008) developed a solution framework which integrates evolutionary algorithm with multi-objective computing budget allocation method (MOCBA). They employed MOCBA to efficiently allocate simulation replications to solutions in the current population. The proposed approach is applied on a multi-objective aircraft spare parts allocation problem to find a set of non-dominated solutions. Horng, Lin, Lee, and Chen (2013) used GA in combination with a surrogate model to find a set of good solutions in the global search stage. In the second stage they employed a probabilistic local search method to identify the approximate local optima. In the final stage OCBA is used to obtain the best solution among the promising ones identified previously.

In this paper, we propose two efficient algorithms (based on GA) that are both theoretically robust and of practical value for solving the DOvS problem with a single stochastic constraint. In both proposed algorithms we use GA to guide the search process, because it is a population-based algorithm that simultaneously considers multiple candidate solutions and is shown to be more robust to stochastic noise (Xu et al., 2010). GA works with a *population* of potential solutions and moves this population toward the optimum iteratively. The terms *iteration* and *generation* are used interchangeably in this work to refer to the process of transforming one population of solutions to another. The proposed GA is adapted to handle two performance measures with stochastic noise (i.e., a stochastic objective and constraint) in a simulation environment. Two types of R&S procedure are incorporated into our DOvS algorithms to enhance its statistical efficiency and validity. We use FDP (see Andradóttir & Kim (2010)) repeatedly in the proposed GA to ensure that the candidate solutions in a population are feasible with respect to the stochastic constraint (with some confidence). See Appendix A.1 for the statistical guarantee provided by FDP. At the end of the algorithms we also invoke the clean-up procedure proposed in Boesel, Nelson, and Ishii (2003) to select the best with respect to the stochastic objective from a set of potential solutions. See Appendix A.2 for a detailed description of this clean-up procedure. The first proposed DOvS algorithm guarantees global convergence as the simulation effort goes to infinity (under the condition that the picked solution is feasible), and also guarantees to choose the best among all evaluated possibly feasible solutions with a specified confidence level. Of course it is somewhat reassuring to have convergence statements, in the sense that the algorithms will eventually reach the global optimal when given large enough simulation effort. However, the algorithm's finite-time efficiency may be sacrificed to maintain this theoretically appealing property. Further, this algorithm has to visit every solution infinitely often to guarantee convergence, which is not very practically meaningful especially when the sampling budget is limited. We therefore propose the second DOvS algorithm, which is more heuristic-oriented and can take advantage of the desirable inherent properties of GA (e.g., the adaptive constraint-handling techniques and the mechanism of elite population, see Coello (2002)). The second algorithm is designed to identify the best solution among the final elite population, and may deliver competitive performance in a reasonable computation time.

The paper is organized as follows. In Section 2 we define the DOvS problem with a single stochastic constraint, and introduce the relevant notations and assumptions. Sections 3 and 4 present two DOvS algorithms that adopt different sampling rules and searching mechanisms, and thus provide different statistical guarantees. We give a high-level review of the existing techniques we incorporate, and a detailed description of only the most critical enhancements. An empirical evaluation to compare different algorithms is provided in Section 5, while the paper ends with some concluding remarks in Section 6. The convergence proof and some details of our algorithms are contained in Appendix A.

2. Framework

Our goal is to select the solution with the largest or smallest expected performance (in terms of the stochastic objective) among a large number of candidate solutions that satisfy a single stochastic constraint. Let $G_j(\mathbf{x}_i)$ denote the jth simulation observation taken from solution $\mathbf{x}_i$ (associated with the objective performance measure), and let $H_j(\mathbf{x}_i)$ be the jth simulation observation taken from solution $\mathbf{x}_i$ (associated with the stochastic constraint). The ith solution $\mathbf{x}_i$ is a vector of d integer decision variables in a feasible region $\mathbf{\Omega}$, and is denoted by $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{id}) = \{x_{i\ell}, \ell = 1, 2, \dots, d\}$. The expected performances with respect to the objective and the constraint are defined as $g(\mathbf{x}_i) = E[G_j(\mathbf{x}_i)]$ and $h(\mathbf{x}_i) = E[H_j(\mathbf{x}_i)]$ for $i = 1, 2, \dots, k,\ j = 1, 2, \dots$, respectively. A general formulation of the constrained DOvS problem of interest is described as follows:

$$\min_{\mathbf{x}_i \in \mathbf{\Omega}} g(\mathbf{x}_i),$$

where the feasible region $\mathbf{\Omega}$ is defined by the following stochastic constraint:

$$h(\mathbf{x}_i) \geqslant Q,$$