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Approximate queueing models for capacitated multi-stage inventory systems under base-stock control

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ABSTRACT

A queueing analysis is presented for base-stock controlled multi-stage production-inventory systems with capacity constraints. The exact queueing model is approximated by replacing some state-dependent conditional probabilities (that are used to express the transition rates) by constants. Two recursive algorithms (each with several variants) are developed for analysis of the steady-state performance. It is analytically shown that one of these algorithms is equivalent to the existing approximations given in the literature. The system studied here is more general than the systems studied in the literature. The numerical investigation for three-stage systems shows that the proposed approximations work well to estimate the relevant performance measures.

1. Introduction

In this paper, we analyze multi-stage production-inventory models with a base-stock controlled stock point at each stage. Such models arise within the context of pull-type make-to-stock manufacturing systems. Specifically, we consider a single item type and a serial system with a fixed sequence of manufacturing facilities. The corresponding queueing model is one of the basic building blocks to be investigated for the analysis of multi-item generalizations with various process sequences.

Contribution of the paper is summarized below.

- The approximation introduced by Avşar and Zijm (2003, Chapter 1) for the steady-state performance analysis of two-stage systems is extended in this study to multi-stage systems. We introduce the proposed approximation approach for the case of modeling the manufacturing facilities as single exponential servers. In this case, a single server is considered at each stage. The generalization of our results for replacing single exponential servers with open Jackson networks follows based on the analytical investigation of Avşar and Zijm (2003, Chapter 1) for the same generalization in two-stage systems. This generalization is skipped here for the sake of brevity; the reader is referred to the technical report due to Avşar and Zijm (2009).
- The aforementioned extension is improved by proposing alternative approximations.
  The approximation we consider gives a product-form marginal steady-state distribution for the subsystem consisting of the first two stages. Using this distribution, we come up with a product-form marginal distribution of the subsystem consisting of the second and third stages. Proceeding recursively, a product-form marginal distribution is obtained for every subsystem consisting of two successive stages. We devise two recursive algorithms in this article for two alternative approximate derivations of the steady-state probability distributions.
- The proposed approximations are analytically and numerically compared with the approximations in the literature for exactly the same setting as ours when the manufacturing facilities are modeled as single exponential servers.
  The studies directly related to our work are due to Buzacott, Price, and Shantikumar (1992), Svoronos and Zipkin (1991), Lee and Zipkin (1992, 1995), Duri, Frein, and Di Mascolo (2000) and Gupta and Selvaraju (2006). (A comparative review of these studies is presented in Section 3. The studies of Ettl, Feigin, Lin, & Yao (2000) and Liu, Liu, & Yao (2004) are also among the related literature to be cited here.) These studies are restricted to the case of a single exponential server at each stage whereas our approach lends itself to the generalization of modeling the manufacturing facilities as open Jackson networks.
  We analytically prove that one of the algorithms we propose is equivalent to the existing approximations given by Lee and Zipkin (1992), Buzacott et al. (1992) and Gupta and Selvaraju (2006). Each equivalence relation is for one of three different uses of this algorithm. The other algorithm we give cannot be

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directly compared with the existing approximations.

Our investigation for equivalence of the approximation we propose and that of Lee and Zipkin (1992) implies the aforementioned other equivalence relations due to similar investigations in the related literature. The study of Duri et al. (2000) leads to equivalence of the approximations proposed by Lee and Zipkin (1992) and Buzacott et al. (1992). Also, Gupta and Selvaraju (2006) mainly work with the approximations of Buzacott et al. (1992) for extensions and possible improvements.

Importance of the equivalence relation proved in this study results from (i) using less restrictive assumptions for our approach as compared to that of Lee and Zipkin (1992) and (ii) obtaining product-form steady-state distributions with our approach unlike the matrix-based solution due to Lee and Zipkin (1992). (ii) allows us to consider extensions for more complex systems with open Jackson networks having product-form steady-state distributions when the arrivals are Poisson.

This study focuses on the analytical investigation outlined above. The numerical experiments to compare the alternative approximations considered in this article are limited to three-stage systems.

Another advantage of the approximation approach we use here is its potential to study not only sequential systems but also the following system configurations: two-indenture repairable item systems, base-stock and kanban controlled assembly systems and closed loop two-echelon repairable item systems (see Zijm & Avşar, 2003; Avşar, Zijm, & Rodoplu, 2009; Topan & Avşar, 2011; Spanjers, van Ommeren, & Zijm, 2005).

This paper is organized as follows. In Section 2.1, the queueing model is introduced for the case of single exponential servers at each stage. The approximation and the proposed algorithms are presented in Section 2.2. Section 3 is devoted to the analytical and numerical comparison of the approximate distributions proposed in this paper and the existing ones in the literature. In Section 4, we point out future research directions. The extensions, details and some of the proofs that are skipped in this article for the sake of brevity are given in the technical report due to Avşar and Zijm (2009).

2. Proposed analysis

In this section, a serial queueing model is given for multi-stage production-inventory systems with one manufacturing facility at each stage. The manufacturing facilities are modeled as single exponential servers. There is a base-stock controlled stock point at each stage. This relatively simple setting allows us to introduce the approximation rather easily. The generalization for the case where each stage is a job shop modeled as a product-form network (open Jackson network) is given by Avşar and Zijm (2009).

See Fig. 1 below for the three-stage system with single server facilities. Demands for the finished items stocked at the last stage, stage J, arrive according to a Poisson process with rate λ. A request for an item at any stage triggers the generation of a request for the partially processed item stocked at the preceding stage (in this way demand propagates upstream through the system). Requests that cannot be satisfied immediately are backordered. A synchronization station preceding each stage represents the merge of the request and the demanded item in stock whenever there is at least one of both. For stage j such that $j \in \{1, \ldots, J-1\}$, the synchronization is to feed the succeeding stages and for stage J it is to directly satisfy the customer demand. Stock control policies employed are of base-stock type with base-stock level S_j for stage $j = 1, \ldots, J$ and there are S_j units in stock initially. The facilities are operated under the first-come-first-served discipline and there is no limit on the size of the queues. Typically, managers are interested in expected lead times, work-in-process inventories and customer fill rates, as a function of base stock levels and production capacities (processing rates).

Define random variables N_j and $\overline{N}_j$ as the number of items to be served or being served at stage j with exponential service rate μ_j and the number of items in stock at stage j, respectively. Random variable K_j denotes the number of backordered requests at stage j. For $N_j = n_j, \overline{N}_j = \bar{n}_j, K_j = k_j$, the following inventory balance equations hold due to the base-stock inventory control policies under which the system is operated: $n_1 + \bar{n}_1 - k_1 = S_1$, and $(n_j + k_{j-1}) + \bar{n}_j - k_j = S_j$ for $j = 2, \ldots, J$. Synchronization at stage j implies the following relation: $\bar{n}_j \cdot k_j = 0$ for $j = 1, \ldots, J$. It is assumed that $\rho_j = \lambda/\mu_j < 1$ for $j = 1, \ldots, J$.

2.1. Model

Since the two-stage system is a special case, $J = 2$, of the extension considered in this article for multi-stage systems, we start this section with a review of the model due to Avşar and Zijm (2003, Chapter 1) for two-stage systems. This model is extended for three-stage systems and, then, the extension is generalized for multi-stage systems.

2.1.1. Review for two-stage system

Based on the inventory balance equations and the properties of the synchronization at each stage, it is easy to show that a full state description for the system under consideration is given by $(n_1, \ldots, n_J)$. For $J = 2$ with state description (n_1, n_2), the transition diagram of the queueing model is given in Fig. 2 below. The solution of the balance equations for the transitions in Fig. 2 would give the steady-state probability distribution for (n_1, n_2), i.e., $P(N_1 = n_1, N_2 = n_2)$. Unfortunately, this solution is not easy to characterize analytically; therefore, the following aggregation of the state space is considered. All (n_1, n_2) pairs with $n_1 \leqslant S_1$ are aggregated into $(k_1, n_2) = (0, n_2)$ while each (n_1, n_2) pair with $n_1 = S_1 + k_1$ for $k_1 > 0$ corresponds to (k_1, n_2). This aggregation is called a partial aggregation because only a part of the states with $n_1 \leqslant S_1$ are aggregated. Then,

$$\widetilde{P}(K_1 = k_1, N_2 = n_2) = \begin{cases} \sum_{n_1 \leqslant S_1} P(N_1 = n_1, N_2 = n_2) & \text{for } k_1 = 0, \\ P(N_1 = S_1 + k_1, N_2 = n_2) & \text{for } k_1 > 0, \end{cases}$$

which is implied by the inventory balance equation for stage 1. By $\sum_{n \leqslant S} P(N = n)$, we indicate the summation of $P(N = n)$ over all n such that $n \leqslant S$. Aggregation of the model in Fig. 2 leads to the transition diagram in Fig. 3 for $j = 1$. Here, some transition rates are expressed in terms of conditional probabilities $q_1(n_2)$ and $r_1(k_1, n_2)$ defined for $k_1 = 0$ and $k_1 > 0$, respectively. For the formulations and definitions of these conditional probabilities, see Definition 1 and Remark 1 by setting $j = 1$.

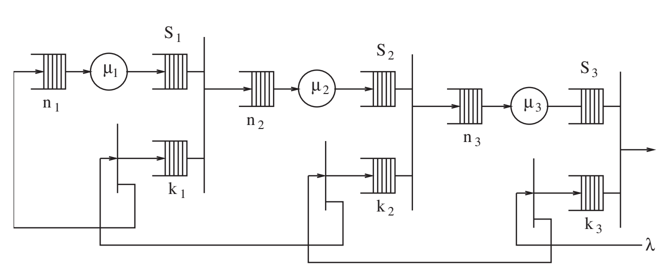

Fig. 1. Three-stage model.