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# Using phase-type models to cost stroke patient care across health, social and community services

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ABSTRACT

Stroke disease places a heavy burden on society, incurring long periods of time in hospital and community care, and associated costs. Also stroke is a highly complex disease with diverse outcomes and multiple strategies for therapy and care. Previously a modeling framework has been developed which clusters patients into classes with respect to their length of stay (LOS) in hospital. Phase-type models were then used to describe patient flows for each cluster. Also multiple outcomes, such as discharge to normal residence, nursing home, or death can be permitted. We here add costs to this model and obtain the Moment Generating Function for the total cost of a system consisting of multiple transient phase-type classes with multiple absorbing states. This system represents different classes of patients in different hospital and community services states. Based on stroke patients' data from the Belfast City Hospital, various scenarios are explored with a focus on comparing the cost of thrombolysis treatment under different regimes. The overall modeling framework characterizes the behavior of stroke patient populations, with a focus on integrated system-wide costing and planning, encompassing hospital and community services. Within this general framework we have developed models which take account of patient heterogeneity and multiple care options. Such complex strategies depend crucially on developing a deep engagement with the health care professionals and underpinning the models with detailed patient-specific data.


## 1. Introduction

Stroke disease, as the single biggest cause of disability in the United Kingdom (UK), is a huge drain on the public purse and, as such, is of particular concern to policy makers. The total societal cost of stroke in the UK has been estimated at nearly £9 billion per year (Saka, McGuire, & Wolfe, 2009) of which 50% is accounted for by direct care costs, including hospitalization, outpatient and GP visits, drug costs and community or nursing home care. 27% of the cost is accounted for by informal care costs from family, friends or nonNHS professionals, and the remainder by indirect costs to the economy due to premature death or disablement. It is estimated that 25% of post stroke patients in the UK enter institutional care (Sackley & Pound, 2002). In addition, on-going care such as long-term nursing and community based care place substantial costs on health and social services (Sundberg, Bagust, & Terent, 2003). Our focus in this paper is on extending a previous Markov modeling framework (McClean, Barton, Garg, & Fullerton, 2011) that developed analytic models to describe lengths of stay (LOS) of stroke patients in various components of hospital, social and community care. The aim is to decrease patient delays, efficiently use resources and improve adherence to government targets. Our current work extends the framework to add costs thus facilitating economic evaluation to assess alternative clinical and care strategies.

Economic evaluation of health care commonly uses cost models to compare health care strategies, and provide solutions which produce health care benefits for the patient and cost-efficient strategies for the health care provider. Such approaches are particularly important with regard to providing information that facilitates the efficient and fair allocation of scarce resources (Cooper, Abrams, Sutton, Turner, & Lambert, 2003).

A number of modeling solutions have been utilized in such contexts, principally macro-economic models which are generally population or cohort based, typically regression, or micro-economic models which are individual based, such as Decision Trees or Markov models. Such models may be analytic (Gillespie et al., 2011) or employ discrete event simulation (DES) (Katsaliaki & Mustafee, 2011), although the former may be limited in terms of detail (McClean et al., 2011) while the latter may have high computational

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load (Hiligsmann et al., 2009). Cooper, Brailsford, and Davies (2007) have identified the choice of modeling technique to depend on "the acceptance of the modeling technique, model error, model appropriateness, dimensionality and ease and speed of model development". In particular they argue that Markov models are well suited to simple chronic interventions. The use of Markov models for health care economic evaluation has also been advocated as a way of handling both costs and outcomes simultaneously in a simple and intuitive manner (Briggs & Sculpher, 1998).

Although Markov models have been criticized in the past in terms of limited assumptions they can be readily extended in various ways to overcome such limitations. For example, the basic assumptions of the Markov model, such as the Markov property can be relaxed within an analytic framework such as a semi-Markov model (Papadopoulou, Tsaklides, Mcclean, & Garg, 2012). This approach is highly flexible and has the ability to provide a realistic representation of complexity and variability commonly found in heterogeneous health care systems.

A key issue which must be addressed within a framework for modeling and costing patient pathways is the heterogeneity of patient pathways and LOS characteristics. Such heterogeneity arises from a number of sources e.g. method of admission, diagnosis, severity of illness, age, gender, treatment (Dimakou, Parkin, Devlin, & Appleby, 2009; Faddy & McClean, 2005; Garg, McClean, Barton, Meenan, & Fullerton, 2012; Harper, 2002; Marshall & McClean, 2004). A number of approaches have therefore been used to cluster LOS data and generate patient groups, some of these have been discussed in McClean et al. (2011). We here build on previous work and base this clustering on survival analysis, although there are a number of other approaches to identifying groups of patients that have similar pathways; costing then becomes specific to the particular pathway a patient is following.

Our main objective is to extend a previous modeling framework, with particular applicability to stroke patient care, which associates costs with patient pathways, based on covariates such as age, gender, or diagnosis, using a phase-type distribution. We also encompass multiple outcomes, such as discharge to normal residence, nursing home, or death. We develop an analytic framework for economic modeling using routinely available data. The approach is illustrated using data for stroke patients originally admitted to the Belfast City Hospital (BCH).

## 2. The analytic framework

The analytical cost model for stroke planning is based on Markov phase-type models, where mathematical results are obtained for a basic scenario. Phase-type models are becoming increasingly popular for health care applications (Fackrell, 2009; Harper, Knight, & Marshall, 2012; Knight & Harper, 2012). This work is an extension of the model developed by McClean et al. (2011), and will extend it to include costs. Thus the analytic model can allow us to easily implement and quickly evaluate basic changes for different hospital settings.

Coxian phase-type distributions have been previously shown to give a good fit to hospital LOS data,e.g., (Faddy & McClean, 2005; Marshall & McClean, 2004). They are (in our case, continuous time) Markov models that are intuitively appealing as we can think of the patient as progressing through various phases of hospital, social care and community care such as acute, treatment, rehabilitation and long stay (Fig. 1).

From the technical point of view, the advantages of using the Coxian phase-type distribution to describe LOS in hospital are (i) their mathematical tractability; (ii) parsimonious parameterization – a general phase-type representation requires a large number of parameters, with associated difficulties in estimation; and (iii) any positively supported distribution can be approximated by a phase-type distribution with an appropriate number of parameters; however, the order of the representation may be large. McClean et al., 2011 developed a framework that classifies patient stays based on identifying homogeneous groups in terms of their LOS distributions; different admission probabilities therefore pertain to different classes. Classes are characterized using appropriate covariates, in this case: gender, age, diagnosis and outcome. Patients in the various classes follow separate pathways, with corresponding different admission probabilities for each class. Another feature of this framework is that, unlike previous work, it allows for a number of absorbing states – for example, these might be the patient's normal residence, private nursing home or death. Such an approach allows community care to be modeled as well as hospital states thus describing the integrated system of stroke patient care, rather than sub-systems of the overall care process.

In this paper we extend this methodology to the Moment Generating Function (MGF) for total cost of a whole integrated care system. In Subsection 2.1 we mathematically describe how Coxian phase-type models can be used to model LOS in hospital, before defining the MGF of LOS in hospital in Subsection 2.2. In the following subsections we will build on this result to define the MGF of hospital and community care, and the MGF ofa system with multiple transient classes and multiple absorbing states. The MGFs are defined in Sections 2.2, 2.3 and 2.4, but are proven in Appendices A, B and C.

### 2.1. The MGF of LOS in hospital

In general, we use Coxian phase type models to describe duration as a $k$ state continuous time Markov model where the absorbing state $S_{k+1}$ represents the event death or discharge of the patient (Fig. 1). A patient can be admitted to the system only via the first state. Transitions are possible from any transient state $S_i$ ($i = 1,2,\ldots,k-1$) to the next state $S_{i+1}$ with transition rate $\lambda_i$. Also transitions are possible from any state $S_i$ to the absorbing state $S_{k+1}$ with transition rate $\mu_i$ (Fig. 1).

The infinitesimal transition matrix $\mathbf{Q}$, which consists of the transition rates between the different states, is defined as:

$$\mathbf{Q} = \begin{pmatrix} -(\lambda_1+\mu_1) & \lambda_1 & 0 & \ldots & 0 \\ 0 & -(\lambda_2+\mu_2) & \lambda_2 & \ldots & 0 \\ . & . & . & \ldots & . \\ 0 & 0 & 0 & -(\lambda_{k-1}+\mu_{k-1}) & \lambda_{k-1} \\ & & & 0 & -\mu_k \end{pmatrix}$$

and the absorption matrix, which consists of the transitions from the transient states to the absorbing states, is defined as $\mathbf{q} = (\mu_1,\ldots,\mu_k)'$.

The time spent in the hospital before death or discharge has the probability density function

$$f(t) = \mathbf{p}\exp(\mathbf{Q}t)\mathbf{q}$$

where the row vector $\mathbf{p} = \{p_i\}$ is the initial probability distribution and, for the Coxian model, is defined as $\mathbf{p} = (1,\ldots,0)$, and $\exp(\mathbf{A}) = \mathbf{I} + \mathbf{A} + \mathbf{A}^2/2! + \ldots$ for any matrix $\mathbf{A}$.

We also note that, by differentiating the series expansion ($\exp(\mathbf{Q}t)$), we obtain:

$$\frac{d}{dt}(\exp(\mathbf{Q}t)) = (\exp(\mathbf{Q}t))\mathbf{Q}.$$

Also, $-\mathbf{Q}^{-1}\mathbf{q} = \mathbf{e}$, where $\mathbf{e} = (1,\ldots,1)'$ is a $k \times 1$ column vector.

For a non-defective phase-type distribution, starting from any transient state (phase), absorption occurs with probability 1 (Latouche & Ramaswami, 1999, Theorem 2.4.2). The matrix $\mathbf{P}(t) = \{P_{ij}(t)\} = \exp(\mathbf{Q}t)$ is the matrix of transition probabilities from