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Decision Support

Dynamic pricing with uncertain production cost: An alternating-move approach

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ABSTRACT

This article studies a two-firm dynamic pricing model with random production costs. The firms produce the same perishable products over an infinite time horizon when production (or operation) costs are random. In each period, each firm determines its price and production levels based on its current production cost and its opponent's previous price level. We use an alternating-move game to model this problem and show that there exists a unique subgame perfect Nash equilibrium in production and pricing decisions. We provide a closed-form solution for the firm's pricing policy. Finally, we study the game in the case of incomplete information, when both or one of the firms do not have access to the current prices charged by their opponents.

1. Introduction

A considerable number of papers in the operations literature study the problem of using innovative pricing and inventory control in order to maximize expected profit. Different approaches such as dynamic pricing, revenue management, inventory control, joint inventory and pricing control, and joint production and pricing control have been used to achieve this goal (see for example Gallego & van Ryzin (1994, 1997), Chatwin (2000), Feng & Xiao (2000), Lin (2004), Zhao & Zheng (2000), Hall, Kopalle, & Pyke (2008), Fleischmann, Hall, & Pyke (2006), Lin & Sibdari (2009), Sibdari & Pyke (2010)).

Most of these studies consider a single retailer who enjoys monopoly power. An important assumption of these models is that the demand (or the distribution of the random demand) depends only on the price (or inventory level in the case of inventory control) set by the monopolist. This assumption is not quite realistic since customers typically can compare price and quality of substitutable products and choose the most appropriate one (note that some products such steel, oil, and airline seats do not differ appreciably in quality).

One approach to the multiple-firm problem is to employ game theory. Recently, quite a few papers have appeared in the supply chain and revenue management literatures that use game theory to tackle incentive-related problems with multiple agents. The game theory literature in general may be categorized by *simultaneous-move* and *alternating-move* games. In simultaneous-move games, the players make decisions at the same time without observing each other's decisions, while in alternating-move games, the players take turns with their decisions and observe the last action taken by other players. To best of our knowledge, most of the related papers in supply chain and revenue management literature use simultaneous-move games. The majority of these papers, furthermore, use stationary policies in which the outcome of a single-stage game is applied for the entire time horizon.

A major assumption of simultaneous-move games is that the parameters of the model, such as production costs, are deterministic and remain constant. In reality, however, different internal parameters such as production or labor costs (e.g. pension plan cost) may not be constant. Variable production costs may lead firms to adjust their price levels even if the opponents' price levels are constant. These unexpected changes in price levels force other firms to react and adjust their prices accordingly, even if their production costs remain the same. As a result, in such environments, the firms do not make decisions at the same time, nor can they ignore actions taken by their opponents in previous periods. Addressing this observation is the motivation of our study.

We consider a two-firm alternating-move dynamic game over an infinite time horizon. We make some simplifying assumptions for purposes of tractability. Nevertheless, we derive generalizable insights, and we anticipate that this research will stimulate further work that will relax some of the assumptions. Specifically, we assume that the firms produce similar products that can instantly be produced with no production limits. The production cost of each firm over the time horizon is random and is only realized at the time of production. The firms take turns making pricing and

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production moves (or equivalently, decisions). When a firm (say firm 1) moves in a period (say odd periods), the other firm (firm 2) moves in the next period (even periods). Furthermore, neither firm updates its own strategy in the periods in which the other firm moves. A firm determines its price and production levels based on its own production cost and the price charged by the other firm in the previous period in order to maximize its discounted expected profit. We show the existence of a unique Nash equilibrium and provide a procedure to calculate the equilibrium price and production levels.

The assumption that firms engage in alternating moves might raise the question of why firms follow this restriction rather than moving simultaneously. In some cases, of course, reality exactly mirrors an alternating-move game. The Coke–Pepsi example shown in Fig. 1, for instance, captures data from a particular market and supermarket chain and clearly illustrates an alternating move scenario (Kopalle, 2007). New product introductions may also create alternating pricing moves. For instance, recent competitive responses to the iPhone were priced roughly at existing iPhone prices. However, the subsequent introduction the iPhone 3G S led Apple to price previous models as low as $99, creating a difficult pricing problem for Apple's competitors. We anticipate a similar competition between Amazon's Kindle and related products. In these cases, competition may spawn prices suggestive of an alternating move game.

Note that if the firms choose prices simultaneously, an adverse price shock might actually increase the equilibrium price levels, thereby decreasing aggregate demand. This outcome can arise in a simultaneous move game since each firm has an incentive to deviate based on the other firm's behavior. To illustrate this argument, in Section 4, we provide a numerical study to show how a firm can lower its profit by ignoring the last action taken by the other firm. Specifically, we use a numerical study with two firms when firm 2 considers the last action taken by firm 1, while firm 1 behaves as if both firms use a simultaneous move game. Maskin and Tirole (1988a, 1988b) also argue that the nature of a game may differ with respect to the timing patterns. They provide a timing game by the firms and show a more elaborate class of models where firms move at any time they choose. The details of this timing game are beyond the scope of our research and we refer interested readers to these references.

Many industries exhibit other characteristics of our model. For instance, in the airline industry, fuel cost is uncertain, and pricing decisions of airlines not only depend on competition and market demand, but also on fuel cost. A change in fuel cost faced by one airline may force that airline to change its ticket prices accordingly. In response, other airlines, which might face different costs, set new price levels. In the past 10 months (at this writing), the cost of a barrel of crude oil rose to $140, then fell to $42, only to rebound to around $70. Some airlines hedged fuel costs and thus were relatively insulated from these changes, while other airlines' costs fluctuated wildly.

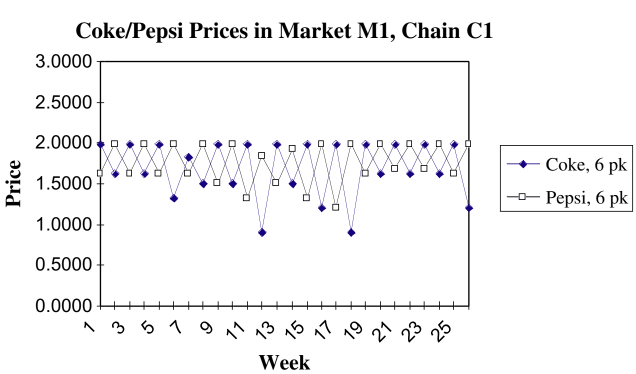

Fig. 1. Alternating price game between Coke and Pepsi.

The deep recession of course had an impact on market demand, further complicating the picture. Ticket prices varied accordingly, with US coast-to-coast fares falling at one point as low as $99. An interesting observation is that even Southwest Airlines, which is renowned for its hedging policies and industry-leading low prices, was forced to respond to competitors' low prices when fuel prices plummeted. Production cost for airlines of course includes capacity, along with the costs of fuel, labor, and other components. Some of these costs, such as capacity and labor, are sunk in the near term. However, even capacity may change when fuel prices change. A sharp increase in fuel price will induce an airline to make changes to routes, capacity, and airplanes (and therefore number of available seats). Furthermore, airlines that hedge do not always hedge their total fuel consumption. There have been times in recent history that Southwest was significantly hedged on fuel cost, but even then it may have only been hedged to about 60% of its consumption. Therefore, all airlines face short term fluctuations in fuel price. They have had mixed ability to pass through these costs to the consumer. On international routes, airlines often can pass through 100% of a fuel price increase, but not on shorter haul domestic routes. So ticket prices, for a given airline, will change depending on the route, the amount of hedging they have employed, and of course, competition. For a detailed analysis see Carter, Rogers, and Simkins (2007) and Ubhi (1996).

In other industries, such as high technology and consumer electronics, firms source critical components from low cost countries. In recent months, as the RMB has strengthened against the dollar, and wage rates in China have increased dramatically, the cost of sourcing components from China has risen, implying that competitors who source from, say, Mexico now enjoy lower relative costs. The latter firm may adjust prices, forcing the former firm to respond.

In all these cases, we observe alternating-move scenarios and/or uncertain costs, where our alternating-move, random cost model is more suitable than the existing simultaneous-move models. Furthermore, each of these examples shares fundamental characteristics of our model. For instance, a major assumption of our paper is that the products are perishable and that firms do not carry them across periods, and thus inventory levels do not have an impact on firms' pricing decisions. This assumption is valid in the airline industry example. Airlines, on the other hand, do not have infinite capacity. High tech products, consumer electronics, and soft drinks manufacturers have much more control over capacity, but inventory is less perishable than airline seats. Our goal is to further the literature on the intersection of pricing and operations with tractable models, specifically assuming infinite capacity and products that are perishable. Relaxing the assumptions of infinite capacity and of perishable products would be valueble extensions for our paper. In the rest of this paper, in order to be consistent with the literature, we use the term production cost instead of operation cost.

A number of recent papers in service and production management have employed game theoretic tools to address incentive related problems in presence of multiple players. Often models in economics and management science use price, production, or inventory level as the strategic decision factor. Examples of research papers considering price as a decision variable include Bernstein and Federguen (2003, 2004) and Kirman and Sobel (1974), and Lin and Sibdari (2009). For a detailed survey on this topic see Cachon and Netessine (2004). Research papers that address competition between retailers in the context of supply chain management often assume that firms choose both price and inventory levels. Recent papers addressing this problem include Cachon and