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Decision Support

Portfolio insurance: Gap risk under conditional multiples ☆

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ABSTRACT

The research on financial portfolio optimization has been originally developed by Markowitz (1952). It has been further extended in many directions, among them the portfolio insurance theory introduced by Leland and Rubinstein (1976) for the "Option Based Portfolio Insurance" (OBPI) and Perold (1986) for the "Constant Proportion Portfolio Insurance" method (CPPI). The recent financial crisis has dramatically emphasized the interest of such portfolio strategies. This paper examines the CPPI method when the multiple is allowed to vary over time. To control the risk of such portfolio management, a quantile approach is introduced together with expected shortfall criteria. In this framework, we provide explicit upper bounds on the multiple as function of past asset returns and volatilities. These values can be statistically estimated from financial data, using for example ARCH type models. We show how the multiple can be chosen in order to satisfy the guarantee condition, at a given level of probability and for various financial market conditions.

1. Introduction

Portfolio selection theory has been originally introduced by Markowitz (1952) and further developed in many directions (see e.g. Merton (1971, 1990), for the continuous-time setting). Various decision criteria such as the expected utility theory can be considered to determine the optimal portfolios (see e.g. Campbell & Viceira, 2002; Prigent, 2007; Yu, Pang, Troutt, & Hou, 2009). The impact of portfolio rebalancing methods has been also investigated (see e.g. Detemple, Garcia, & Rindishbacher, 2003; Wang & Forsyth, 2011; Yu & Lee, 2011). However, investors often search for additional guarantees (portfolio insurance), in particular when financial markets drop. The financial management industry extensively uses portfolio insurance methods for various financial instruments: equities, bonds, structured credit products, hedge funds, etc.

Portfolio insurance has two main objectives: first, to allow investors to recover at maturity at least a given percentage of their initial investments (usually 100%); second, to benefit from potential financial market rises. Thus, it allows investors to limit downside risk in bearish financial markets, second to get significant portfolio returns in bullish markets. Such portfolio management is dramatically relevant during financial crises. The two main portfolio insurance methods are: the Option Based Portfolio Insurance (OBPI), introduced by Leland and Rubinstein (1976); the Constant Proportion Portfolio Insurance (CPPI) considered by Perold (1986) for fixed-income instruments and Black and Jones (1987) for equity instruments (see also Perold & Sharpe, 1988). As regards OBPI method, the portfolio is invested in a risky benchmark asset covered by a put option written on it. The strike of the option is equal to a fixed proportion of the initially invested amount (which corresponds to the capital insured at maturity).

The CPPI method is based on a dynamic portfolio allocation along the whole management period. The investor determines a floor which is defined as the lowest acceptable portfolio value. Then, she invests an amount ("the exposure") in the risky asset, which corresponds to a given proportion (called "the multiple") of the excess of the portfolio value over the floor (this difference is usually called "the cushion"). The remaining funds are usually invested in cash (treasury-bills for example). Both the management parameters (floor and multiple) depend on the investor's risk tolerance. The CPPI strategy implies that the exposure is about zero if the cushion value is near zero. In continuous-time, this property prevents portfolio value from falling below the floor, except if a very sharp drop in the market occurs before the investor can modify her portfolio allocation. Some of the properties of portfolio insurance have been previously studied by Black and Rouhani (1989) and Black and Perold (1992) when the risky asset follows a geometric Brownian motion (GBM) and by Bertrand and Prigent (2003) when the volatility is stochastic. Comparisons between standard portfolio insurance methods are illustrated by Bookstaber

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and Langsam (2000), Cesari and Cremonini (2003) and Bertrand and Prigent (2005, 2011). The main conclusion is that it is not so easy to rank these strategies, except by their sensitivity Vega to the volatility of the risky asset. However, the CPPI method is the best strategy when the market drops or increases by a significant amount.[1]

The main issue of the CPPI strategy is to choose the crucial parameter which determines the portfolio risk exposure, known as the multiple (denoted by m). Note that financial institutions directly bear the risk of the insured portfolios they sell: at maturity, if the portfolio value is smaller than the guaranteed floor ("the gap risk"), they must compensate the corresponding loss with their own capitals. Thus, one crucial question for the financial institution that promotes such funds is: what exposure to the risky asset or, equivalently, what level of the multiple m to accept? On the one hand, since the portfolio expected return is increasing with respect to the multiple m, customers want the multiple as high as possible. On the other hand, due to market imperfections,[2] portfolio managers must control the gap risk by setting the multiple m smaller than an upper bound. If a maximal daily historical drop (e.g. -20%) is anticipated during the time period, the portfolio manager chooses $m \leqslant 5$.[3] It implies low expected portfolio returns. If the risky asset drop is assumed to be less significant (e.g. -10%), the upper bound can be chosen higher (e.g. $m \leqslant 10$). If the portfolio manager wants to use higher multiple values, she can base the strategy on quantile hedging. In that case, the multiple can be chosen as high as possible but so that the portfolio value will always be above the floor at a given probability level (typically 99%).

The answer to this latter issue has important practical implications. However, the usual assumption in the literature is that the multiple is constant through time. It implies that: it has to be determined initially; it is unconditional, meaning that, whatever the market and portfolio value fluctuations, the risk exposure is always equal to the same proportion of the cushion. Thus, such portfolio strategy does not take sufficiently account of the possibility that the risky asset drops, which challenges the concept of dynamic portfolio insurance. In a discrete-time setting, the extreme value approach has been applied to the standard CPPI method with constant multiple by Bertrand and Prigent (2002) to control the gap risk. Balder, Brandl, and Mahayni (2009) have analyzed CPPI effectiveness using quantile conditions when the risky asset follows a GBM that is discretized at deterministic times. Cont and Tankov (2009) have examined CPPI strategies for exponential Lévy processes. But all these unconditional methods reduce the risk exposure to a constant risky asset exposure, which cannot be dynamically adjusted.

In this paper, we introduce another CPPI method, directly linked to a risk management approach and based on a conditional multiple. In this setting, we determine upper bounds on the conditional time-varying multiple, using both quantile ("Value-at-Risk") and expected shortfall criteria to control gap risks. Such downside risk measures have been introduced in the late nineties and further analyzed (see e.g. Pedersen & Satchell, 1998; Artzner, Delbaen, Eber, & Heath, 1999; Szegö, 2002). They are related to economical capital allocation as recommended by Basel II for banking laws and regulations (see Goovaerts, Kaas, & Dhaene, 2002). They have also been also intensively used in portfolio management (see Rockafellar & Uryasev, 2002). Unlike Hamidi, Jurczenko, and Maillet (2009) and Hamidi, Maillet, and Prigent (2009) who use conditional autoregressive Value-at-Risk to estimate gap risks, we consider a quite general parametric model based on Autoregressive Conditionally Heteroscedastic (ARCH) type return modelling. In this framework, we succeed in identifying exactly the various upper bounds. Our results prove that a conditional multiple can be determined as functions of state variables. These latter ones are usually the past stock logreturns and volatilities.

The remainder of the paper is organized as follows. Section 2 presents the basic properties of the CPPI model. Section 3 introduces the modified CPPI method with a conditional multiple, based on various quantile and expected shortfall conditions. In particular, upper bounds on the multiple are provided and analyzed. Section 4 illustrates numerically the previous theoretical results and provides a detailed description of the portfolio return distributions. Some of the proofs are gathered in Appendix.

2. The standard CPPI model

2.1. The financial market

Basically, two assets are involved: the riskless asset, B, with deterministic rates (usually Treasury bills or other liquid money market instruments) and the risky one, S (usually a market index, equity index for instance). Changes in asset prices are supposed to occur at discrete times $(t_k)_{1\leqslant k\leqslant n}$ along a whole management period $[0, T]$.

The riskless asset evolves according to deterministic rates[4] denoted by r_{t_k} for the time periods $[t_{k-1}, t_k]$ (it means that $B_{t_k} = B_{t_{k-1}}(1 + r_{t_k})$). The variations of the stock price S between two times t_{k-1} and t_k are defined by:

$$\Delta S_{t_k} = S_{t_k} - S_{t_{k-1}}.$$

Since we search an upper bound on the multiple m, we have to focus on the left hand side of the probability distribution of $\frac{\Delta S_{t_k}}{S_{t_{k-1}}}$. Thus, we introduce the following notation:

$$X_{t_k} = -\frac{\Delta S_{t_k}}{S_{t_{k-1}}} = \frac{S_{t_{k-1}} - S_{t_k}}{S_{t_{k-1}}},$$

where X_{t_k} denotes the opposite of the relative jump of the risky asset at time t_k. In fact, when we want to determine an upper bound on the multiple m, we have only to consider positive values of X.

Denote by M_T, the maximum of the finite sequence $(X_{t_k})_{1\leqslant k\leqslant n}$. We have:

$$M_T = Max(X_{t_1}, \ldots, X_{t_n}). \tag{1}$$

2.2. The standard CPPI portfolio

Usually, the individual, having initially invested an amount V_0, wants to recover a fixed percentage p of V_0 at a given maturity T. To provide a terminal portfolio value V_T higher than the insured amount pV_0, the portfolio manager keeps the portfolio value V_{t_k} above the floor $F_{t_k} = F_{t_{k-1}} e^{-r_{t_k}(t_k - t_{k-1})}$ at any time t_k during the management period $[0, T]$ and with $F_0 = p \cdot V_0 \cdot e^{-\sum r_{t_k}(t_k - t_{k-1})}$ (for fixed interest rates $r_{t_k} = r$, we get: $F_0 = p \cdot V_0 \cdot e^{-rT}$). For this purpose, she determines:

[1] Note also that, using various stochastic dominance (SD) criteria up to third order and assuming that the risky underlying asset follows a GBM, Zagst and Kraus (2011) provide very specific parameter conditions implying the second- and third-order SD of the CPPI strategy.

[2] For example, portfolio managers cannot actually rebalance portfolios in continuous time.

[3] See Proposition 1 for details about determination of this upper bound.

[4] Note that, since the rebalancing frequencies $(t_k - t_{k-1})$ are usually small, the standard levels of r_{t_k} have no significant impact on the numerical values of the upper bounds provided in what follows. Additionally, if the interest rate is stochastic, it is mainly its mean that determines the floor: For instance, consider a guaranteed percentage equal to 100% and a time horizon equal to 8 years. Then, if the mean of the interest rate is equal to 2% (resp. 3%), the initial floor value is equal to about 15% (resp. 21%) of the initial portfolio value (see details below in Section 2.2).