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Decision Support

A three-dimensional voting system in Hong Kong

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ABSTRACT

The voting system of the Legislative Council of Hong Kong (Legco) is sometimes unicameral and sometimes bicameral, depending on whether the bill is proposed by the Hong Kong government. Therefore, although without any representative within Legco, the Hong Kong government has certain degree of legislative power – as if there is a virtual representative of the Hong Kong government within the Legco. By introducing such a virtual representative of the Hong Kong government, we show that Legco is a three-dimensional voting system. We also calculate two power indices of the Hong Kong government through this virtual representative and consider the *C*-dimension and the *W*-dimension of Legco. Finally, some implications of this Legco model to the current constitutional reform in Hong Kong will be given.

1. Introduction

The voting system used in the Hong Kong Legislative Council (Legco) is unique within the current, global range of electoral systems because it is sometimes unicameral and sometimes bicameral, depending on whether the bill is proposed by the government of the Hong Kong Special Administrative Region (HKSAR). Because of this special feature, it would be interesting to study this voting system from the mathematical point of view. In this article, we shall construct a mathematical model of this voting system and conduct a detailed mathematical analysis of it. For example, we shall measure the complexity of the voting mechanism of Legco by computing its dimension.

It was proposed in Taylor (1995) that different voting systems can be classified by their dimensions. We shall state the precise definition of the dimension of a voting system in the next section but it is easier to grasp the idea by considering some examples. The usual unicameral system in which the bills will be passed by a simple majority vote of the members is a one dimensional voting system. Bicameral legislatures require a concurrent majority to pass legislation are two dimensional voting systems. In 1995, Alan D. Taylor mentioned in Taylor (1995) that he did not know any real-world voting system of dimension 3 or higher and this point was reiterated in page 255 of the second edition of Taylor (1995). Such a real-world voting system of dimension 3 was first provided by Josep Freixas in 2004. He showed in Freixas (2004) that the dimension of the European Union Council under the Nice rules (since 2000) is 3. In this article, we shall show that the voting system of Legco (since 1998) is also of dimension 3. So we have two very different real voting systems of dimension 3.

By 2014, Legco consists of two groups of legislators: one group comprising 35 members selected in the functional constituencies and the other group comprising the remaining 35 members elected by universal suffrage in the geographical constituencies. About half of the 35 functional constituencies seats go to business sectors, about one-third belongs to sectors for professionals and the rest are for representatives of social organizations or district councils. For a detailed analysis of the functional constituencies, see Loh and Civic Exchange (2006) and Ma (2009).

Since the handover of Hong Kong in 1997, in order to strengthen the executive dominance over the legislature, the Basic Law (the constitutional document for the HKSAR) requires the passage of motions, bill or amendments to government bills introduced by legislators to pass by the concurrent majorities of two groups. On the other hand, motions, amendments to motions, bills and amendments to bill raised by the Hong Kong government only need a simple majority vote of the members present to pass.

Therefore, Legco is sometimes unicameral and sometimes bicameral, depending on whether the bill is proposed by the Hong Kong government. This unique feature of Legco makes the computation of the dimension of it a non-trivial task. For example, given a coalition of 35 members from the geographical constituencies and 2 members from the functional constituencies, there is no way to tell if it is a winning coalition or a losing coalition (unless you know if the bill is proposed by the Hong Kong government). To overcome this difficulty, we introduce a virtual representative of the Hong Kong government, which will vote for a bill if it is proposed by the government but against a bill otherwise. To be more precise,

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the 35 members from geographical constituencies are numbered $1,\ldots,35$, the 35 members from functional constituencies are numbered $36,\ldots,70$, and the virtual member (the government) is numbered 71. Then, a coalition or subset S of the set $\{1,2,\ldots,70,71\}$ is a winning coalition if and only if

(a) $71 \in S$ and $|S \cap \{1,\ldots,70\}| \geqslant 36$, or
(b) $71 \notin S$, $|S \cap \{1,\ldots,35\}| > 18$ and $|S \cap \{36,\ldots,70\}| > 18$.

where $V \cap U$ is the intersection of the two coalitions V and U and $|U|$ is the number of members in the coalition U.

With this mathematical model of Legco, the voting system of Legco becomes a simple game (see the definition in the next section) and it is now possible to compute the dimension of Legco which will turn out to be 3 (see Section 3).

Although there is no representative of the government of HKSAR in Legco, this voting system has given the Hong Kong government a certain degree of legislative power. With the introduction of member 71, the virtual representative of the government, we can then calculate various power indices of the Hong Kong government within Legco and compare its power with that of any individual Legco member. This will be done in Section 4. Finally, in Section 5, we shall discuss some implications of our mathematical analysis of Legco to the current constitutional reform in Hong Kong.

2. Dimension of a simple game

Definition 1. A (monotonic) *simple* game or a voting system is a pair (N, v) where $N = \{1,\ldots,n\}$ is the set of players and $v : 2^N \to \{0,1\}$ is the characteristic function defined on the power set 2^N of N, which satisfies $v(\phi) = 0$, $v(N) = 1$ and $v(S) \leqslant v(T)$ whenever $S \subseteq T$. A coalition of players $S \subseteq N$ is *winning* if $v(S) = 1$ and *losing* if $v(S) = 0$.

A *weighted majority game* is a simple game which can be realized by a vector $(w_1,\ldots,w_n)$ together with a threshold q which makes the representation $[q; w_1,\ldots,w_n]$ in such a way that S is a winning coalition if and only if $\sum_{j \in S} w_j \geqslant q$.

A *weighted m-majority game* is a simple game which can be expressed by m realizations $[q^i; w_1^i,\ldots,w_n^i]$, $1 \leqslant i \leqslant m$, in such a way that S is a winning coalition if and only if $\sum_{j \in S} w_j^i \geqslant q^i$ for all $1 \leqslant i \leqslant m$. So a weighted m-majority game can be considered as an intersection of m weighted majority games and we can also represent it by an amalgamated matrix

$$\begin{pmatrix} q^1; & w_1^1 & \cdots & w_n^1 \\ \vdots & & & \vdots \\ q^m; & w_1^m & \cdots & w_n^m \end{pmatrix}.$$

Every weighted majority game is obviously a (monotonic) simple game. However, the converse is in general not true and we shall see in the next section that the voting system of Legco is not a weighted majority game. To show that certain voting system cannot be realized as a weighted majority game, it would be useful to know that any weighted majority game must be *swap robust*, namely, for any two winning coalitions S and S' in a weighted voting system, if we make a one-for-one exchange of players, then at least one of the two resulting coalitions must still be a winning coalition (see Taylor (1995)). Here one of the players in the swap must belong to S but not S', and the other must belong to S' but not S.

Even though not any simple game is a weighted majority game, it was proved in Taylor (1995) that every simple game can be realized as a weighted m-majority game. The smallest such possible m is called the *dimension* of the game.

The computation of the dimension of a simple game has been proved to be a hard computational problem (see Deineko & Woeginger (2006)), thus our calculation is a complex task which will be done by combinatorial arguments for the particular voting system at hand. The dimension of other complex voting systems using combinatorial arguments were studied quite recently in Freixas and Puente (2008).

However, even if two games are of the same dimension, they may not be equivalent. In the literature, different types of dimensions have been introduced (see Freixas & Marciniak (2009)). Before giving their definitions, we need to introduce two orderings related to the power of individual members in a game (see Carreras & Freixas (2008)).

For any $i,j \in N$, we say that $i \leq_D j$ if for any $U \subset N$ such that $i,j \notin U$, we have $U \cup \{j\}$ is a winning coalition whenever $U \cup \{i\}$ is. A game is *complete* if $\leq_D$ is total. It is known that a game is complete if and only if it is swap robust.

An $i \in N$ is *crucial* in a winning coalition U if $U \setminus \{i\}$ is no longer winning. We say that $i \leq_d j$ if the number of winning coalitions of size k containing i with i crucial is smaller than or equal to the number of those containing j with j crucial for all $1 \leqslant k \leqslant n$. A game is *weakly complete* if $\leq_d$ is total.

Every simple game can be realized as an interaction of a finite number of complete games, the smallest possible number is called the *C-dimension* of the game. Likewise, every simple game can be realized as an interaction of a finite number of weakly complete games, the smallest possible number is called the *W-dimension* of the game. For a detailed analysis of various types of dimensions, we refer the readers to Freixas and Marciniak (2009).

Every weighted game is complete and every complete game is weakly complete (see Carreras & Freixas (2008)). Hence,

1. W-dimension $(v) \leqslant C$-dimension $(v) \leqslant$ dimension (v) for all simple game v;
2. if the simple game v is weighted, then dimension (v) is 1 and hence both W-dimension (v) and C-dimension (v) are equal to 1;
3. if the simple game v is complete, then C-dimension (v) is equal to 1 and therefore W-dimension (v) is also equal to 1.

3. Hong Kong Legco system

Recall that the Legco members are divided into two groups: half of the members are returned by geographical constituencies through direct elections, and the other half by functional constituencies. On the writing of this article, there are 70 members in the current fifth term Legco (2012–2016). The composition of Legco beyond the third term is not specified in the Basic Law (the constitutional document for the Hong Kong Special Administrative Region). However, article 68 of the Basic Law requires that

> "*The method for forming the Legislative Council shall be specified in the light of the actual situation in the Hong Kong Special Administrative Region and in accordance with the principle of gradual and orderly progress. The ultimate aim is the election of all the members of the Legislative Council by universal suffrage.*"

While the Basic Law now no longer expressly dictates the formation of the fifth term Legco of HKSAR in the year 2012, in December 2007, the Standing Committee of National People's Congress (SCNPC) decided that

> "*The ratio of functional constituency members to geographical constituency members shall not be changed and the procedures for voting on bills and motions in the Legislative Council shall remain unchanged.*"