European Journal of Operational Research 236 (2014) 351–360

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

# Profit criteria involving risk in price setting of virtual products

Tatyana Chernonog *, Tal Avinadav

*Department of Management, Bar-Ilan University, Ramat-Gan 52900, Israel*

ARTICLE INFO

*Article history:*
Received 28 November 2012
Accepted 3 December 2013
Available online 11 December 2013

*Keywords:*
Pricing
Multiple criteria
Risk
Supply chain management
Decision analysis

ABSTRACT

This work deals with pricing of "virtual" products, i.e., products that a retailer can supply after demand has been realized. Such products allow the retailer to avoid holding costs and ensure timely fulfillment of demand with no risk of shortage. Demand is commonly price-dependent and uncertain, and we seek to maximize each of three criteria: expected profit, the likelihood of achieving a profit target, and the profit for a given percentile. Simultaneous multiple criteria are also explored. Two forms of demand uncertainty are considered in the analysis: the multiplicative form, where, due to stochastic dominance, all the investigated profit criteria—and, in fact, any utility function of the profit—can be optimized simultaneously; and the additive form, where stochastic dominance cannot occur. Under the multiplicative form of demand, the property of stochastic dominance is shown to hold in a two-echelon supply chain (comprising both the supplier and the retailer) and in a centralized system.



## 1. Introduction

The last decade has seen a rapid increase in the market share and variety of intangible products. This increase is largely a result of the proliferation of digital platforms for content consumption, as well as the development of the internet as a direct channel for delivering goods and transferring payments (Waelbroeck, 2013). Sales of intangible products can generate substantial revenues: For example, a recent article in *Time* magazine noted that in a one-day national promotion, the website LivingSocial grossed more than 11 million dollars from the sales of $20 Amazon.com gift cards, sold for $10 each (Tuttle, 2012). Yet it seems that the development of pricing models has not kept pace with that of the market. Traditional models, formulated primarily for the pricing of tangible products, fail to capture key properties of intangible products and are therefore of limited utility to the digital retailer. In particular, many of the parameters incorporated into traditional models are superfluous for products that are not held in inventory and can be produced (or delivered) in infinite quantities and in a negligible amount of time—after demand has been realized. We refer to such products as "virtual" products. Examples of virtual products include (1) smartphone applications that are sold online; (2) licenses to download and use software for a calendar year (e.g., SAS, Maple); (3) gift cards being sold at a discount online (e.g., http://www.groupon.com/ about); (4) electronic newspapers, books, music, TV programs and movies to be downloaded from the net for a fee; and (5) club/association memberships that are sold for a calendar year (e.g., professional societies).

Seeking to fill the gap in the literature, herein we construct a pricing model for virtual products. We assume that demand is influenced not only by retail price but by many other, more minor, factors and therefore is stochastic in nature. The uncertainty in demand is translated into uncertainty of profits, and the criterion for maximization is therefore a random variable. We suggest that in selecting a criterion for maximization, it is necessary to take into account the decision maker's approach to risk. The most commonly used criterion of profit is the expectation (see, e.g., Dana, 2001; Gerchak, 2012; Granot & Yin, 2008; Van Mieghem & Dada, 1999); the selection of this criterion assumes that the decision maker is risk neutral, i.e., has a linear utility function. Only a non-linear utility function can capture risk considerations with regard to profit. In the absence of a utility function (as discussed later) other profit criteria that do consider risk can be used. Herein we consider three criteria: expectation, the probability of achieving a given target, and the profit for a given percentile.

In addition to proposing a model that is tailored to virtual products, this paper makes three primary contributions. First, we present methods for deriving prices that maximize each of the three profit-related criteria mentioned above. It is possible that none of these criteria, by itself, captures the preferences of the retailer. As a consequence, our second contribution is providing means for the decision maker to select wisely his or her preferred price when considering several profit criteria. This is done by constructing a set of profit criteria values that are not dominated, and identifying the prices that yield these values (referred to as the set of "efficient" prices). Third, this paper shows that under certain assumptions of demand uncertainty, one price optimizes all profit criteria.

* Corresponding author. Tel.: +972 050 7977317.
*E-mail address:* Tatyana.Chernonog@biu.ac.il (T. Chernonog).

0377-2217/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2013.12.004

The remainder of the paper is organized as follows: Section 2 presents the related literature. Section 3 introduces the problem formulation and presents different optimization criteria that may express the retailer's profit preferences. In Section 4, we consider this problem assuming a multiplicative demand form and show that there is an optimal price whose profit is stochastically dominant. In Section 5, we analyze the additive demand form, for which we show that stochastic dominance of profits does not exist. We analyze the retailer's pricing decision under various optimization criteria and perform a sensitivity analysis. Then we provide a means to establish a non-dominated set of criteria values that can aid the retailer in selecting a preferred price. We illustrate this technique using numerical examples. Section 6 provides two extensions of the multiplicative demand case: a two-echelon supply chain and limited capacity. We conclude in Section 7 with implications and directions for future work.

## 2. Literature review

We review three streams of related literature. The first is a well-established research stream comprising price-setting models under uncertain demand, whose objective is to maximize profits. An example for such a model is the "price-setting newsvendor problem" (see review in Petruzzi & Dada (1999), and recently in Xing, Wang, & Liu (2012), Xu & Lu (2013)), where both price and order quantity are to be determined simultaneously. However, this model is not well suited to virtual products, as it considers overage and underage costs, which are not relevant for these products. Additional models in this stream include peak-load pricing models in the field of economics (see review in Crew, Fernando, & Kleindorfer, 1995) and product postponement strategy models in field of operations research (see, for example, Cheng, Li, Wan, & Wang, 2010; Granot & Yin, 2008; Reimann, 2012; Van Mieghem & Dada, 1999). These two types of models share the following assumptions: (i) demand decreases in price; (ii) demand uncertainty has either a multiplicative or additive form; (iii) capacity cost is considered; and (iv) production capacity is limited. Assumptions (iii) and (iv) are superfluous for virtual products. In the model we develop for virtual products, we adopt assumptions (i) and (ii) and relax assumptions (iii) and (iv). In addition, whereas much of the literature in this stream assumes a specific price-dependent demand function and only one objective function (the profit expectation in most cases), our model enables a broader analysis, as it considers various decision criteria under a general demand function with uncertainty.

Managers and firms are often risk-sensitive (usually risk-averse). As a result, when demand is uncertain and the range of profits is not small, an approach of maximizing expected profit may not represent their preferences. An alternative approach is to rely on normative utility theory, which considers risk factors by assessing the subjective utility function of the firms and maximizing its expected value (see Leland, 1972). However, the utility theory approach is not without disadvantages: first, it is not easy to extract the utility function; second, the objective function of expected utility may not be easy to analyze. Furthermore, behavioral economists, beginning with Allais (1953) and including, most prominently, Tversky and Kahneman (1974), have questioned the assumptions underlying this theory. Non-expected utility models have been proposed to capture expected utility anomalies, yet these approaches have not mitigated the disadvantages associated with utility theory (for a review, see Starmer, 2000).

Therefore, we refer to a second stream of literature, comprising analytic studies in which the goal is to achieve "satisfactory" outcomes rather than maximum expected profit. Extensive research (e.g., Brown & Tang, 2006; Lanzillotti, 1958; Merchant & Manzoni, 1989; Morris & Fuller, 1989) suggests that firm managers indeed seek to achieve such objectives, termed "satisficing" by Simon (1959). Shi, Zhao, and Xia (2010) suggest that satisficing objectives have a major advantage over expected-profit maximization. The advantage is that they are "more practical for many individuals and firms. E.g., in the business world, it is common that individuals and firms are rewarded if they can meet or exceed some preset profit targets." There are two main satisficing objectives, which are based, respectively, on probabilistic and percentile measures of risk (see Uryasev, 2000). The first seeks to maximize the probability to achieve a target profit (see Abbas & Matheson, 2005; Bordley & Kirkwood, 2004; Bordley & LiCalzi, 2000; He & Khouja, 2011; Lau, 1980; Parlar & Weng, 2003; Shi et al., 2010; Shi, Yan, Xi, & Zhao, 2011). The second seeks to maximize a percentile (i.e., quantile) criterion (see Delage & Mannor, 2010; Filar, Krass, & Ross, 1995; Henig, 1990). Each of these objectives actually replaces the utility function.

The third research stream relates to studies in multi-criteria decision analysis. According to Babalos, Philippas, Doumpos, and Zopounidis (2012), "multi-criteria decision making provides an arsenal of techniques for aggregating multiple criteria in performance evaluation problems in order to select, rank, classify, and describe a set of alternative options". Belton and Stewart (2002), Gandibleux (2002) and Ehrgott (2005) review the main concepts, principles and techniques in this field, and recent advances and research trends are presented in Ehrgott, Figueira, and Greco (2010), Zopounidis and Pardalos (2010) and Zopounidis, Fabozzi, and Doumpos (2014). In our paper, given a group of criteria, we identify a set of non-dominated profit criteria values and the prices that yield these values (referred to as the set of "efficient" prices) and investigate their properties.

## 3. Problem formulation

Consider a retailer who wishes to choose a unit retail price $p$ for a product or a service (from now on, 'product' refers also to services). The retailer can pull any quantity of units he wishes from the product's supplier at a wholesale price $w$ per unit. The business process is composed of three sequential stages as illustrated in Fig. 1. First, $p$ is determined; then demand is revealed; and finally, the retailer orders the quantity demanded in the previous stage and delivers the units to the customers.

The demand for the product is a random variable, denoted by $\tilde{D}(p)$, whose expected value $D(p)$ is a decreasing function of the retail price, $p$. Let $\varepsilon$ be a random variable that is price-independent, and let $F_\varepsilon(x)$ be its cumulative distribution function (CDF). We analyze the two most common forms (see, for example, Petruzzi & Dada, 1999; Qin, Wang, Vakharia, Chen, & Seref, 2011) of stochastic demand: (i) the multiplicative form: $\tilde{D}(p) = D(p)\varepsilon,\ E[\varepsilon] = 1, \varepsilon \geqslant 0$ and (ii) the additive form: $\tilde{D}(p) = D(p) + \varepsilon,\ E[\varepsilon] = 0$. These forms are specific cases of a general demand function $\tilde{D}(p)$, where $\tilde{D}(p_1)$ stochastically dominates $\tilde{D}(p_2)$ for $p_1 < p_2$. Two cases of stochastic dominance are useful here (Whitmore & Findlay,

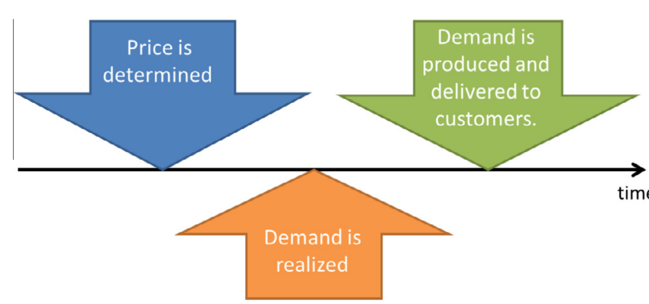


**Fig. 1.** Sequence of events.