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Discrete Optimization

Equilibrium network design of shared-vehicle systems

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ABSTRACT

An equilibrium network design model is formulated to determine the optimal configuration of a vehicle sharing program (VSP). A VSP involves a fleet of vehicles (bicycles, cars, or electric vehicles) positioned strategically across a network. In a flexible VSP, users are permitted to check out vehicles to perform trips and return the vehicles to stations close to their destinations. VSP operators need to determine an optimal configuration in terms of station locations, vehicle inventories, and station capacities, that maximizes revenue. Since users are likely to use the VSP resources only if their travel utilities improve, a generalized equilibrium based approach is adopted to design the system. The model takes the form of a bi-level, mixed-integer program. Model properties of uniqueness, inefficiency of equilibrium, and transformations that lead to an exact solution approach are presented. Computational tests on several synthetic instances demonstrate the nature of the equilibrium configuration, the trade-offs between operator and user objectives, and insights for deploying such systems.

1. Setting

In recent years, vehicle sharing programs (VSPs) have garnered interest of urban communities across the world as an economical and sustainable solution for urban mobility. In their most flexible form, VSPs allow users to check out vehicles (bicycles, electric vehicles, or cars) close to their origins and drop them off at stations close to their destinations. For short trips, the shared-vehicle serves as an individual modal alternative. For longer trips, it serves as a vital leg of an intermodal route. For transit-based intermodal routes, VSPs increase travel utility by improving flexibility, offering greater departure time choice, and increasing transit accessibility. These improvements can serve to increase the adoption of transit in the United States (US), where the automobile is the dominant mode of transport (USDOT, 2001).

Shared-vehicle systems are increasingly prevalent in urban areas across the world. In 2010, bicycle sharing programs were estimated to exist in 125 cities with an additional 45 programs being planned (Shaheen, Guzman, & Zhang, 2010), while car sharing services exist in an estimated 1000 cities. In 2012, the number of existing and planned bicycle sharing systems is estimated at 480 (Meddin & DeMaio, 2012). From the supply side, private stakeholders and public agencies have recognized the value of these systems as the use of technology has mitigated the risk of theft. Users have found shared-vehicles to be profitable. Velib, the pioneering bicycle sharing system in Paris with 1450 stations and 20,000 bicycles, reports around 80,000 trips per day, with ridership as high as 120,000 (Erlanger, 2008) and an estimated two million trips within the first two years of operation (DeMaio, 2009). The Capital bikeshare system in Washington, DC with 114 stations and 1000 bicycles completed 1.4 million trips in its first year.

This paper addresses the system design of flexible VSPs from the perspective of the VSP operator. In practice, the organizational structure behind the design and operation of VSPs varies considerably for different cities. The term 'VSP operator' is designated herein as the entity responsible for designing the VSP system. The primary stakeholders involved with the design tasks can include public transportation agencies, public transit authorities, non-profit organizations, private for-profit companies, local communities and advertising companies. Transportation agencies typically initiate the design process by mandating operating standards. These standards can pertain to location of VSP stations, shared-vehicle inventories, integration of payment systems, and desired level-of-service. The agency may work with external service providers (non-profit entities, commercial providers, advertising agencies) to build out the system and operate it. These external service providers are paid through revenue-sharing agreements with the locality or through other means, such as advertising rights at VSP stations and on vehicles. Another business model involves private companies solely determining the configuration of the system, choosing to provide services where they may be most utilized. Thus, given the myriad forms in which the organizational roles can be structured, the VSP operator can either be a public agency or a private participator.

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Regardless of the entity that provides the vehicle-sharing service, users utilize the system only if it reduces their travel disutility. The shared-vehicle system is an attractive individual modal alternative when shared-vehicles are available near the origin and destination of the user. For longer trips, the shared-vehicle can serve as a 'last-mile' connection for existing transit services. In this case, the urban transportation network can be viewed as a hierarchical network with the VSP serving as the feeder service to a transit backbone (see Fig. 1).

VSP operators must decide (1) where to locate stations, (2) the number of vehicle docking slots to provide, and (3) the initial number of vehicles to place at each station. An equilibrium network design model is proposed herein that optimally determines a VSP configuration, defined by location, capacity, and inventory at stations. The model recognizes the operator's lack of control over the utilization of the system, since the usage of the VSP is driven by decisions on the part of patrons who seek to minimize their travel disutilities. As shared vehicles could be potentially used for longer trips, the model explicitly incorporates existing transit services to allow for intermodal paths. At the upper level, the VSP operator determines the optimal configuration of the system (supply). At the lower level, users respond to the VSP configuration and optimize their personal itineraries (demand). The VSP operator in turn adjusts the VSP configuration to maximize ridership. At equilibrium, the optimal configuration is one that supports travelers who derive utility from using the shared-vehicle to complete trips. Additionally, the VSP operator acts first to establish the service offerings, while users act as *followers*. This leader–follower framework results in a Stackelberg equilibrium that is modeled as a bi-level program. Since bi-level programs generally are nonconvex and non-differentiable, specialized techniques are needed that make the problem tractable. When the follower's problem is convex, known transformation techniques can be exploited to convert the model to a single-level program that is convex.

The resulting large mixed-integer program (MIP) can be solved using existing solvers.

The literature on shared-vehicle systems is growing as these systems are more widely adopted (Shaheen et al., 2010). Existing works deal with several aspects pertaining to the analysis, evaluation, design and operation of VSPs. Earlier studies focused on the qualitative characteristics that aim at determining the feasibility (Friedman, 1972; TRB, 2005a) and market potential based on surveys and demographic information (Katzev, 2003; Krykewycz, Puchalsky, Rocks, Bonnette, & Jaskiewicz, 2010; Shaheen, Meyn, & Wipyewski, 2003), and trip forecasts (Kek, Cheu, & Xu, 2004).

From a strategic perspective, system design has received little attention. Awasthi, Breuil, Chauhan, Parent, and Reveillere (2007, 2008) present a multi-criteria decision-making tool based on the analytic hierarchy process (AHP). Their framework relies on experts ranking stations based on various criteria, including land use, population density, vehicle ownership, transit access, and presence of target groups. Lin, Yang, and Chang (2013), Lin and Yang (2011) present a strategic design, where there is explicit consideration for level-of-service of users under stochastic utilization parameters. In addition to determining the location, the model computes flows that a station will handle along with bicycle paths that need to be built. However, the decisions of users and system planners are conflated to form a single objective function for optimizing the design of the system. This approach can be viewed as the system-optimal utilization of the shared-vehicle resources. In practice, however, users employ the system in a myopic fashion and employ a shared-vehicle trip only if it reduces their personal travel disutility, i.e. a user-optimal solution. The user-optimal solution recognizes the lack of control that system operators have over the vehicle fleet, making this distinction critical. Correia and Antunes (2012) present an optimization model that determines depot locations for car-sharing schemes. Their objective is to maximize profit for the operator considering operational costs of relocating the fleet to balance flows. Trip requests are known, and three operational schemes are explored, one where all trips are service, another where operators determine with trips to service, and a hybrid scheme which is conditional on availability. Martinez, Caetano, Eiró, and Cruz (2012) present a mixed-integer program that includes operational dimensions within the more strategic design model. The objective is to maximize net revenue. User preferences are estimated at the zone level using discrete choice models. Candidate sites are determined using a *p*-median problem formulation. In contrast, the presented approach explicitly considers user preferences and intermodalism in the design.

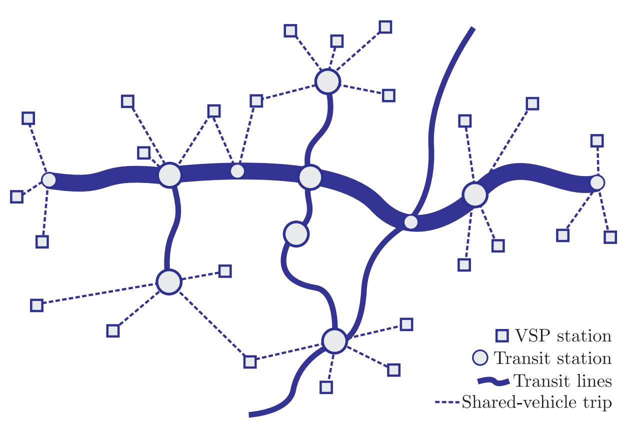

Fig. 1. Hierarchical transit-VSP network.

There has been some research interest in the operational characteristics of shared-vehicles. The authors (Nair & Miller-Hooks, 2011b; Nair, Miller-Hooks, Hampshire, & Bušić, 2012) have developed stochastic optimization models to generate management strategies that ensure system operations at a given reliability and show, for real-world systems, how fleet balancing can be performed to correct short-term demand asymmetry. Barth and Todd (1999) propose a fleet management strategy that shifts the burden to users who perform relocations, while other works have studied relocation issues and formulated decision support tools for the operators (Kek, Cheu, & Chor, 2006, 2009; Kek & Cheu, 2005). More recent work includes a static repositioning model using a user dissatisfaction function (Raviv et al., working paper) and inclusion of routing decisions of service vehicles that perform the redistribution. The latter problem is a variant of the pickup and drop off problem (see recent reviews by Berbeglia, Cordeau, Gribkovskaia, & Laporte, 2007, 2010, for static & dynamic versions). Due to the combinatorial nature of routing decisions, these models can be significantly more challenging to solve for real-world systems. From the machine learning perspective, there have been research efforts aimed at establishing patterns of system utilization (Kaltenbrunner, Meza, Grivolla, Codina, & Banchs, 2010), measurement (Froehlich, Neumann, & Oliver, 2008), and sensing and prediction (Froehlich, Neumann, & Oliver, 2009).

Another pertinent aspect of the literature is the integration of transit with other modes for improving mobility. Several works have studied the role of 'feeder' modes to transit lines (Rodier et al., 2004; Shaheen & Rodier, 2006; Shaheen, Rodier, & Seelig, 2005; TRB, 2005b). The main focus of these studies has been in integrating bicycles and other slow modes with transit through the use of technology to provide better accessibility.

The proposed model is strongly aligned with the access network design problem, which arises in other sectors, such as telecommunications (Balakrishnan, Magnanti, Shulman, & Wong, 1991; Berger & Raghavan, 2004), computer networks (Patrick, 1977), and