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Discrete Optimization

The distance constrained multiple vehicle traveling purchaser problem

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ABSTRACT

In the Distance Constrained Multiple Vehicle Traveling Purchaser Problem (DC-MVTPP) a fleet of vehicles is available to visit suppliers offering products at different prices and with different quantity availabilities. The DC-MVTPP consists in selecting a subset of suppliers so to satisfy products demand at the minimum traveling and purchasing costs, while ensuring that the distance traveled by each vehicle does not exceed a predefined upper bound. The problem generalizes the classical Traveling Purchaser Problem (TPP) and adds new realistic features to the decision problem. In this paper we present different mathematical programming formulations for the problem. A branch-and-price algorithm is also proposed to solve a set partitioning formulation where columns represent feasible routes for the vehicles. At each node of the branch-and-bound tree, the linear relaxation of the set partitioning formulation, augmented by the branching constraints, is solved through column generation. The pricing problem is solved using dynamic programming. A set of instances has been derived from benchmark instances for the asymmetric TPP. Instances with up to 100 suppliers and 200 products have been solved to optimality.

1. Introduction

In many business environment, as those involved in raw materials and components purchase, the selection of suppliers is a key procurement decision. Different aspects influence this decision and different contributions appeared in the literature where purchasing costs are optimized assuming that demand is either deterministic or stochastic. Interested readers are referred to Benton (1991) for a procurement problem with quantity discounts, to Rosenblatt, Herer, and Hefter (1998) for policies on supplier selection, purchase frequency and quantity setting, Chauhan and Proth (2003) for a procurement problem with concave purchase cost and to Zhang and Zhang (2011) for an extension of the traditional setting to include holding and shortage costs, a fixed cost for each selected supplier as well as stochastic demand. More recently, extensions of the so called Total Quantity Discount Problem (TQDP) introduced in Benton (1991) are investigated by Goossens, Maas, Spieksma, and Van de Klundert (2007) who develop an exact method for the problem. Finally, Manerba and Mansini (2012) study the generalization of TQDP where the quantity offered for each product by a supplier is limited. They introduce different valid inequalities for a formulation of the problem and use them in a branch-and-cut algorithm.

Interestingly, all the cited contributions mainly focus on the pricing aspect of the procurement problem. Nevertheless, procurement costs are not just determined by purchasing costs. Typically, transportation cost is a substantial component of the procurement costs that needs to be optimized as well. A procurement setting that explicitly incorporates both purchasing and transportation costs has been studied in Mansini, Savelsbergh, and Tocchella (2012). The authors consider the case of a company that has to select a set of suppliers offering discounts based on the total quantity purchased. The transportation costs are modeled as truckload shipping costs, and thus depend on the total quantity purchased as well.

The transportation cost structure described in Mansini et al. (2012) is appropriate when the transportation service is outsourced to a carrier company. In this paper, we study a procurement setting where the purchaser company needs specified quantities of a variety of products from a set of suppliers and is involved in the direct collection of the purchased products with a fleet of vehicles based at a common depot. Each supplier offers a subset of products at possibly different prices and having different availabilities. The company has to select a set of suppliers and construct a set of routes so that total traveling and purchasing costs are minimized. A distance constraint is set on the route traveled by each vehicle. Such a constraint is determined by the working time of vehicle drivers and imposes that the length of the route (defined in terms of mileage or time) must not exceed a predefined bound. The distinction between mileage and time is irrelevant if all vehicles in the fleet are assumed to travel at the same average speed.

Let $K := \{1, \ldots, n\}$ be the set of products to be purchased, let $M := \{1, \ldots, m\}$ be the set of suppliers to choose from, and a depot

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indexed by 0, and let $F := \{1, \ldots, l\}$ be the fleet of identical vehicles available for the service. Each product $k, k \in K$, can be purchased at a subset $M_k \subseteq M$ of suppliers at a non-negative price p_{ki}, $i \in M_k$. A discrete demand d_k is specified for each product $k \in K$, and a product availability $q_{ki} > 0$ is defined for each product $k \in K$ and each supplier $i \in M_k$ such that $\sum_{i \in M_k} q_{ki} \geqslant d_k$. A distance bound C_{max} is imposed on each vehicle route. Let $G = (V, A)$ be a directed graph where $V := M \cup \{0\}$ is the node set and $A := \{(i, j)$: $i \neq j$, $i, j \in V\}$ is the arc set. We indicate as c_{ij} the cost (distance) of traveling from node i to node j. Each route starts and ends at node 0. The problem looks for a set of routes visiting a subset of nodes in such a way that the total traveling and purchasing costs are minimized while satisfying products demand and the distance bound on each route.

The problem generalizes the well-known Asymmetric Traveling Purchaser Problem (see Riera-Ledesma & Salazar-Gonzalez (2006)) since a fleet, instead of a single vehicle, is available to visit suppliers. Due to the distance bound associated with each vehicle and the fleet of vehicles, we call this generalization the Distance Constrained Multiple Vehicle Traveling Purchaser Problem (DC-MVTPP).

Following the literature on the TPP we refer to the special case with unlimited supplies, i.e. $q_{ki} \geqslant d_k$ for all $k \in K$ and $i \in M_k$, as the *unrestricted* DC-MVTPP, and to the more general case as *restricted* DC-MVTPP. Notice that the unrestricted case is equivalent to assuming that $d_k = 1$ and $q_{ki} = 1$ for all $k \in K$ and $i \in M_k$.

On one hand, the TPP and its variants have been exhaustively studied in the literature. See, for instance, Laporte, Riera-Ledesma, and Salazar-Gonzalez (2003), and more recently, Mansini and Tocchella (2009) for the TPP with budget constraint, Angelelli, Mansini, and Vindigni (2009) and Angelelli, Mansini, and Vindigni (2011) for a dynamic version with quantities decreasing over time, Angelelli, Mansini, and Vindigni (submitted for publication) for a dynamic and stochastic variant, Gouveia, Paias, and Voss (2011) for the TPP with additional side-constraints, and Cambazard and Penz (2012) for a constraint programming solution method to the problem. On the other hand, very few contributions can be found where the TPP is generalized to the multiple vehicle case. In Choi and Lee (2011) the authors introduce a reliability optimization problem as variant of the multi-vehicle TPP. In Riera-Ledesma and Salazar-Gonzalez (2012) the authors analyze the generalization of the asymmetric unrestricted TPP to the multiple vehicle case with capacity constraint. The problem is described as a location routing problem in the context of school bus routing where each student can be seen as a product available at any bus stop he/she can reach by walking. A non-negative cost is associated with the assignment of a student to a stop. This cost can be seen as the distance walked by the student to reach the stop from home. The aim of the problem is to assign each student to a stop so that the total length of routes plus the total assignment cost is minimized while guaranteeing that the number of students assigned to the stops of a route does not exceed the vehicle capacity. The authors propose a model based on a single commodity flow formulation and provide valid inequalities to strengthen its linear programming (LP) relaxation. A branch-and-cut algorithm is also introduced and tested on a large set of randomly generated instances. Then, very recently, the same authors proposed in Riera-Ledesma and Salazar-Gonzalez (2013) a branch-and-cut-and-price approach to address a generalization of the problem described in Riera-Ledesma and Salazar-Gonzalez (2012), taking into account bounds not only on the loading capacity but also on other resources.

The DC-MVTPP is NP-hard, as, other than the TPP, it generalizes also the Distance Constrained VRP (DC-VRP). Indeed, any DC-VRP instance can be solved as an unrestricted DC-MVTPP instance where each supplier offers a product that is not available from the remaining ones, and all products have to be purchased. To the best of our knowledge the problem has never been studied before. Nevertheless, it finds application in different relevant contexts. In addition to the procurement domain already described, the unrestricted DC-MVTPP can also be seen as a variant of the school bus routing problem described in Riera-Ledesma and Salazar-Gonzalez (2012), where instead of a limit on the capacity a time threshold is imposed on each vehicle route corresponding to the hard time window associated with school entrance.

The aim of this paper is to analyze the DC-MVTPP comparing different problem formulations requiring a polynomial number of constraints and to propose and test a branch-and-price approach, based on column generation, for the solution of a set partitioning formulation. The branch-and-price approach is compared with the solution found by the polynomial size formulations when solved with a state-of-the-art commercial solver.

The paper is organized as follows. In Section 2 we present different mathematical formulations of the problem, including a multi-commodity flow formulation, two different single commodity flow formulations, a three-index formulation using the Miller-Tucker-Zemlin generalization of subtour elimination constraints, and a set partitioning formulation. Section 3 is devoted to the description of the *branch-and-price* algorithm for the solution of the set partitioning formulation. Variables, representing feasible routes with respect to the distance bound, are dynamically generated. At each node of the branch-and-bound tree, while solving the LP relaxation of the problem, columns are priced out by means of a label setting algorithm addressing a Shortest Path Problem with Resource Constraints (SPPRC). Routes are imposed to be elementary at the set partitioning model level. An effective restricted master heuristic is used to prune the tree.

We tested the branch-and-price algorithm and the best performing polynomial size formulations solved with CPLEX on a new set of benchmark instances derived from those proposed in Riera-Ledesma and Salazar-Gonzalez (2006) for the asymmetric TPP (ATPP). For each ATPP instance, four DC-MVTPP instances have been generated in such a way that the average number of vehicles required in the solution of the ith instance, $i = 1, 2, 3$, is 2^i. Section 4 reports all the computational results and shows the effectiveness of the proposed solution approaches. We have been able to optimally solve asymmetric instances both restricted and unrestricted with up to 100 suppliers and 200 products, setting a maximum running time of one hour. Notably, the optimality gap is very small for all unsolved instances. We notice that the Miller-Tucker-Zemlin formulation is more effective than the multi-commodity flow formulation allowing CPLEX to massively use internal heuristics and to quickly converge towards good feasible solutions when the number of vehicles is low. The multi-commodity flow formulation, due to its large number of variables, usually takes a lot of time even to find a feasible solution. On the contrary, the single commodity flow formulations may result to be a better compromise in practice, allowing to get more quickly good feasible solutions. In Section 5 final considerations are drawn.

2. Mathematical formulations

In this section we analyze alternative formulations for the DC-MVTPP. We start with a three-index vehicle flow formulation. Then we introduce different polynomial size formulations including a multi-commodity and two single commodity flow formulations, as well as a three-index formulation using a Miller-Tucker-Zemlin (MTZ) generalization of subtour elimination constraints. Finally, we terminate the section with a set partitioning formulation.

2.1. The three-index vehicle flow formulation

The three-index vehicle flow formulation uses $O(m^2|F|)$ binary variables x to indicate whether an arc is traversed in a feasible