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Affine model of inflation-indexed derivatives and inflation risk premium

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ABSTRACT

This paper proposes an affine-based approach which jointly captures the nominal interest rate, the real interest rate, and the inflation risk premium to price inflation-indexed derivatives, including zero-coupon inflation-indexed swaps, year-on-year inflation-indexed swaps, inflation-indexed swaptions, and inflation-indexed caps and floors. We provide an example and explain how to use traded zero-coupon inflation-indexed swap rates to estimate inflation risk premiums.

1. Introduction

Inflation modeling is of great interest in financial research. The volatile oil and commodity prices before the financial crisis in 2008 led to a wave of inflation investing. After the crisis, expanding monetary policies have brought the global economy into another financial turmoil and are expected to push inflation uncertainties to a higher level. For example, Warren Buffet, the CEO of Berkshire Hathaway, worried on May 2, 2010, about the prospect for ”significant inflation” in the United States and elsewhere.

Many financial innovations that were created have been linked to inflation. For example, a futures contract linked to CPI index started trading at the Chicago Board of Trade (CBOT) in 2004. High borrowing needs combined with the desire to improve liquidity has resulted in a surge in the issuance of Treasury Inflation Protected Securities (TIPS) in 1997. In addition, the increasing volatility of inflation rates has introduced inflation as the newest class of assets.[1] In the over-the-counter (OTC) market, there are a variety of inflation-linked derivatives issued. Among these products, the inflation-indexed swap is one of the most popular instruments for financial institutions to hedge the inflation risk in their portfolios. The benefits of the inflation-indexed swap are the exemption of liquidity premium due to its high trading volume and that it is only linked to inflation risk. TIPS, on the other hand, are related to both inflation risk and real interest rate risk.[2] From these perspectives, the inflation-indexed swap is not only a good choice for hedging inflation risk, but also a good instrument for estimating real interest rate and inflation risk premium.

The investor’s perception about inflation risk can be gauged from traded inflation-linked assets. To properly estimate the real interest rate and market-based inflation risk premium, a valuation model considering the inflation risk premium is necessary. Therefore, the main purpose of this paper is to propose an affine model which jointly captures the nominal interest rate, the real interest rate, the inflation expectation, and the inflation risk premium to price inflation-indexed derivatives, including zero-coupon inflation-indexed swaps, year-on-year inflation-indexed swaps, zero-coupon inflation-indexed swaptions, year-on-year inflation-indexed swaptions, and inflation-indexed caps and floors. We also take zero-coupon inflation-indexed swaps as an example that shows how to use the market prices of inflation derivatives to estimate both the real interest rate and inflation risk premiums. In this exercise, we perform two different estimations: one model-free and the other a model-based Kalman Filter, to retrieve the model-implied real interest rate and inflation risk premiums.

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[1] About the detail introduction of inflation derivatives, please refer to Deacon, Derry, and Mirfendereski (2004).

[2] Fleckenstein, Longstaff, and Lustig (in press) provides a replicating portfolio which is composed of inflation swaps and Nominal Treasury Bonds to replicate the payoffs of TIPS. They confirms that the illiquidity of TIPS market causes a significant extra issuing cost compared with the replicating portfolio.

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To estimate inflation rates, the relations between the term structure of nominal interest rates, real interest rates, and inflation rates need to be specified correctly.[3] However, due to the latent property of real interest rates, correctly estimating them is challenging for both macroeconomists and financial economists. In addition, conventional Fisher's hypothesis assumes that the difference between the real and nominal interest rate is only the expected inflation rate, ignoring the uncertainties about future inflation rates. More and more empirical evidences from previous researches show that the Fisher's hypothesis may not be valid, asserting the difference between the real and nominal interest rate is composed of the expected inflation rate and inflation risk premium. Furthermore, the independence of real interest rate and inflation rate is not supported by empirical evidences. Therefore, a correct specification coping with these two features is essential to construct pricing models for inflation-linked products.

In literature, there are a few studies addressing pricing inflation-linked derivatives. Jarrow and Yildirim (2003) introduce the foreign-currency analogy and model nominal interest rates and real interest rates as domestic and foreign assets respectively, using a three-factor HJM model to price TIPS and inflation-indexed options. Mercurio (2005) applies the LIBOR market model to study the inflation-indexed swaps, inflation-indexed caps and floors pricing. In addition, Mercurio and Moreni (2006) further derive closed-form formulas of inflation-indexed caplets and floorlets under the assumption of stochastic volatility. Recently, Hinnerich (2008) proposes an extended HJM framework, allowing for both jumps and stochastic volatility for a market consisting of a money market account, zero-coupon bonds and indexed zero-coupon bonds that are based on a non-traded index. However, none of these models consider the inflation risk premium into modeling; they ignore the inflation risk premium by assuming that Fisher's equation holds.[4]

Therefore, to the best of our knowledge, this paper is the first one to provide a valuation framework which considers inflation risk premium in pricing inflation-linked derivatives. This paper also uses zero-coupon inflation swap rates and provides an example by using a model-free approach and a model-based Kalman-Filter estimation to estimate real interest rate and inflation risk premium.

We organize the remainder of this paper as follows: In Section 2, we introduce the affine model proposed by D'Amico, Kim, and Wei (2009) and extend this model to Section 3 where we derive pricing formulas for inflation-indexed derivatives. In Section 4, we demonstrate how to implement our proposed model by taking a zero-coupon inflation-indexed swap as an example and use a simple model-free approach and a Kalman-Filter estimation approach to estimate the real interest rate and inflation risk premium. We conclude the paper in Section 5.

2. The model

Affine models have been widely used in fixed income modeling due to their flexibility in capturing the time-varying risk premium (see Eraker (2008)). Previous researches also use an affine model to estimate the real bond yield and inflation risk premium, e.g.: Campbell and Viceira (2001), D'Amico et al. (2009). In this paper, we adopt D'Amico et al. (2009)'s model, which is based on the specification of Dai and Singleton (2000) and Duffie, Pan, and Singleton (2000), and assumes that the economic system is driven by a Gaussian vector of latent state variables such that the nominal bond yield, real bond yield, and expected inflation are all affine functions of the latent state vector. By specifying the dynamics of latent state vector, we can explicitly model the time-varying risk premium as well as the correlation between the real bond yield, nominal bond yield, and expected inflation. In this section we first state the affine model for the nominal interest rate, the inflation level, and the real interest rate respectively. We then use this model as the basis to derive the valuation formulas for inflation-linked derivatives. In the following subsections, we introduce the definition of inflation risk premium and its relation to Fisher's Equation. Later, we introduce the affine model used in valuation and model estimation.

2.1. Fisher's equation and inflation risk premium

In Fisher's hypothesis, the relation between the nominal yield and the real yield can be represented as

$$y^N_{t,T_M} = y^R_{t,T_M} + i^e_{t,T_M} \tag{1}$$

where y^N_{t,T_M} is the nominal yield during $[t, T_M]$, and y^R_{t,T_M} is the real yield during $[t, T_M]$. i^e_{t,T_M} is the expected inflation between $[t, T_M]$, equivalent to $1/(T_M - t)E^{\mathbf{P}^N}_t\left[\log\left(I_{T_M}/I_t\right)\right]$, and $E^{\mathbf{P}^N}_t(\cdot)$ is the expectation under nominal physical measure $\mathbf{P}^N$ given information at time t. Based on Fisher's equation, the difference between y^N_{t,T_M} and y^R_{t,T_M} is known as *breakeven inflation* and is the benchmark for a Central Bank to control inflation. However, the breakeven inflation is not only composed of expected inflation, but also the inflation risk premium required by investors. We can derive the relation between y^N_{t,T_M} and y^R_{t,T_M} based on the stochastic discount factor approach by defining a nominal pricing kernel at time t, M^N_t, and a real pricing kernel at time t, M^R_t, where $M^N_t = M^R_t / I_t$. In this framework, the price of a T_M-maturity nominal zero-coupon bond and the real zero-coupon bond at time t can be expressed respectively as:

$$P_N(t, T_M) = E^{\mathbf{P}^N}_t\left(\frac{M^N_{T_M}}{M^N_t}\right)$$

$$P_R(t, T_M) = E^{\mathbf{P}^N}_t\left(\frac{M^R_{T_M}}{M^R_t}\right)$$

With these assumptions, the relation between the nominal yield and the real yield can be expressed as[5]:

$$y^N_{t,T_M} = y^R_{t,T_M} + i^e_{t,T_M} + \phi^I_{t,T_M} \tag{2}$$

where ϕ^I_{t,T_M} is the inflation risk premium which is composed of covariance effect, c_{t,T_M}, and Jensen's effect, J_{t,T_M}. These two effects can be expressed as

$$c_{t,T_M} \equiv -\left(\frac{1}{T_M - t}\right)\log\left[1 + \frac{cov_t\left(\frac{M^R_{T_M}}{M^R_t}, \frac{I_t}{I_{T_M}}\right)}{E^{\mathbf{P}^N}_t\left(\frac{M^R_{T_M}}{M^R_t}\right)E^{\mathbf{P}^N}_t\left(\frac{I_t}{I_{T_M}}\right)}\right]$$

$$J_{t,T_M} \equiv -\left(\frac{1}{T_M - t}\right)\left[\log\left(E^{\mathbf{P}^N}_t\left(\frac{I_t}{I_{T_M}}\right)\right) - E^{\mathbf{P}^N}_t\left(\log\left(\frac{I_t}{I_{T_M}}\right)\right)\right]$$

The covariance effect captures the covariance between the real pricing kernel and the inflation between $[t, T_M]$ since the real pricing kernel and the inflation are all stochastic. Jensen's effect reflects the concavity of logarithmic function. The difference between Eqs. (1) and (2) lies in the modeling of inflation risk premium. Fisher's equation assumes that investors do not require a premium to compensate for inflation risk. However, inflation dynamics are stochastic and investors would require an inflation risk premium to compensate when the market is in equilibrium.

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[3] Boero and Torricelli (1996) provide a comparative evaluation of alternative models of the term structure of interest rates. Schmidt (2011) surveys approaches to modeling the term structure of interest rates.

[4] The literature review about estimating inflation risk premium is given in the Internet Appendix. In this paper, we focus on pricing inflation derivatives and its applications.

[5] Similar argument was introduced and proved in D'Amico et al. (2009).