European Journal of Operational Research 235 (2014) 215–224

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Group decision making with expertons and uncertain generalized probabilistic weighted aggregation operators

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ARTICLE INFO

Article history:
Received 4 November 2012
Accepted 5 October 2013
Available online 15 October 2013

Keywords:
Theory of expertons
Probabilistic weighted average
Interval numbers
Group decision making

ABSTRACT

Expertons and uncertain aggregation operators are tools for dealing with imprecise information that can be assessed with interval numbers. This paper introduces the uncertain generalized probabilistic weighted averaging (UGPWA) operator. It is an aggregation operator that unifies the probability and the weighted average in the same formulation considering the degree of importance that each concept has in the aggregation. Moreover, it is able to assess uncertain environments that cannot be assessed with exact numbers but it is possible to use interval numbers. Thus, we can analyze imprecise information considering the minimum and the maximum result that may occur. Further extensions to this approach are presented including the quasi-arithmetic uncertain probabilistic weighted averaging operator and the uncertain generalized probabilistic weighted moving average. We analyze the applicability of this new approach in a group decision making problem by using the theory of expertons in strategic management.

1. Introduction

Aggregation operators are very common in the literature (Beliakov, Pradera, & Calvo, 2007; Xu & Da, 2003; Yager, Kacprzyk, & Beliakov, 2011). They are very useful for assessing the available information in a more efficient way. One of the most common aggregation operators is the weighted average. It aggregates the information by giving different degrees of importance to the arguments considered in the problem. It has been used in an astonishingly wide range of applications (Beliakov et al., 2007; Torra, 1997). Another very common aggregation operator is the probabilistic aggregation. It aggregates the information by using probabilities in the analysis. It has also been applied in a lot of applications (Merigó, 2010; Yager, Engemann, & Filev, 1995; Yang, Yang, Liu, & Li, 2013; Yang, 2001).

Another type of aggregation operators that are also becoming very popular in the literature are the generalized aggregation operators (Beliakov et al., 2007). Their main characteristic is that they generalize a wide range of aggregation operators by using generalized and quasi-arithmetic means. Thus, we can include in the same formulation arithmetic aggregations, geometric aggregations or quadratic aggregations. For example, Yager (2004) introduced the generalized ordered weighted averaging (GOWA) operator and Fodor, Marichal, and Roubens (1995) the quasi-arithmetic OWA (Quasi-OWA) operator. Merigó and Gil-Lafuente (2009) extended these approaches by using induced aggregation operators. Merigó and Casanovas (2010, 2011a) developed several extensions by using different types of interval and fuzzy numbers. Zhou and Chen (2010) suggested an extension by using logarithmic aggregation operators and Zhou, Chen, and Liu (2012) by using power aggregation operators. Xu and Wang (2012), Xu and Xia (2011) and Zhao, Xu, Ni, and Liu (2010) introduced several extensions when dealing with intuitionistic information.

Recently, Merigó (2012a) has suggested a new approach that unifies the probability and the weighted average in the same formulation and considering the degree of importance of each concept in the aggregation. He has called it the probabilistic weighted averaging (PWA) operator. Its main advantage is that it can consider subjective and objective information in the same formulation. Thus, it is able to assess the information in a more complete way.

Usually, when dealing with these aggregation operators we assume that the information is clearly known and can be assessed with exact numbers. However, in real world problems, it is not so easy to assess the information because usually it is very complex and affected by different types of uncertainties. Thus, the use of exact numbers is not enough because it provides incomplete information and sometimes this may lead to wrong decisions. Therefore, in order to properly assess the information we need to use other techniques for representing the uncertainty in a more complete way such as the use of interval numbers (Moore, 1966). Its main advantage is that it can consider a wide range of scenarios

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http://dx.doi.org/10.1016/j.ejor.2013.10.011

from the minimum to the maximum. Note that by using interval numbers, the previous aggregation operators are known as the uncertain weighted average (UWA) and the uncertain probabilistic aggregation (UPA).

The aim of this paper is to present the uncertain generalized probabilistic weighted averaging (UGPWA) operator. It is a new aggregation operator that unifies the UWA and the UPA operator in the same formulation taking into account the degree of importance that each concept has in the analysis. Thus, it can represent the information considering subjective and objective perspectives and uncertain environments assessed with interval numbers. Moreover, it also uses generalized means that provide a general framework that includes a wide range of particular cases including the UWA, the UPA, the uncertain generalized weighted average (UGWA), the uncertain generalized probabilistic aggregation (UGPA), the uncertain average (UA), the PWA operator, the uncertain PWA (UPWA) and many others. Note that the main advantage of using uncertain information is that the information can be represented in a more complete way considering the most pessimistic and optimistic scenarios and the most possible ones.

We further generalize this approach by using quasi-arithmetic means, obtaining the uncertain quasi-arithmetic PWA (Quasi-UPWA) operator. It provides a more robust generalization that includes the UGPWA operator as a particular case and many other situations. We also extend this framework by using moving averages (Evans, 2002; Merigó, 2012a; Yager, 2008) in order to represent the information in a dynamic way. Thus, we form the uncertain generalized probabilistic weighted moving average (UGPWMA). Its main advantage is that it can deal with problems to be solved in more than one period of time (or equivalent).

We study the applicability of the new approach in a multi-person decision making problem regarding the selection of strategies by using the theory of expertons (Kaufmann, 1988; Kaufmann & Gil-Aluja, 1993). The theory of expertons extends the concept of probabilistic set (Hirota, 1981) for uncertain environments that can be assessed with interval numbers. The main advantage of this approach is that we can analyze the information of the group in a more complete way considering all the individual opinions and producing a final single result. Thus, by using expertons in group decision making problems, the information becomes more robust because it is assessed by several experts and usually the use of several experts in the analysis leads to better decisions.

This paper is organized as follows. In Section 2 we briefly describe some basic concepts. Section 3 introduces the UGPWA operator and Section 4 analyzes several families. Section 5 presents a generalization by using moving averages. Section 6 develops an application in group decision making with the theory of expertons and Section 7 summarizes the main results of the paper.

2. Preliminaries

In this Section we briefly review some basic concepts regarding the interval numbers, the uncertain generalized weighted average and the PWA operator.

2.1. The interval numbers

The interval numbers (Moore, 1966) are a very useful and simple technique for representing the uncertainty that has been used in a wide range of applications.

The interval numbers can be expressed in different forms. For example, if we assume a 4-tuple (a_1, a_2, a_3, a_4), that is, a quadruplet; we could consider that a_1 and a_4 represents the minimum and the maximum of the interval number, and a_2 and a_3, the interval with the highest probability or possibility, depending on the use we want to give to the interval numbers. Note that $a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4$. If $a_1 = a_2 = a_3 = a_4$, then, the interval number is an exact number; if $a_2 = a_3$, it is a 3-tuple known as triplet; and if $a_1 = a_2$ and $a_3 = a_4$, it is a simple 2-tuple interval number.

In the following, we are going to review some basic interval number operations as follows. Let A and B be two triplets, where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$. Then:

(1) $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.
(2) $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.
(3) $A \times k = (k \times a_1, k \times a_2, k \times a_3)$; for $k > 0$.
(4) $A \times B = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)$; for R^+.
(5) $A \div B = (a_1 \div b_3, a_2 \div b_2, a_3 \div b_1)$; for R^+.

Note that other operations could be studied (Merigó & Casanovas, 2011a; Moore, 1966; Wei, 2009; Xu & Da, 2002) but in this paper we focus on these ones. Note also that we call uncertain aggregation operators, to those operators that use interval numbers in the analysis.

2.2. The uncertain generalized weighted average

The uncertain generalized weighted average (UGWA) is an aggregation operator that generalizes the uncertain weighted average (UWA) operator by using generalized means. Thus, it can be implemented in the same problems where the UWA operator has been used because we can always reduce the UGWA to the UWA. Moreover, it can be used in a wide range of other situations because it also includes geometric, quadratic and harmonic aggregations as particular cases. It is defined as follows.

Definition 1. Let Ω be the set of interval numbers. An UGWA operator of dimension n is a mapping $UGWA$: $\Omega^n \to \Omega$ that has an associated weighting vector W of dimension n with $\sum_{i=1}^{n} \tilde{w}_i = 1$ and $\tilde{w}_i \in [0, 1]$ such that:

$$UGWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\sum_{i=1}^{n} \tilde{w}_i \tilde{a}_i^{\lambda} \right)^{1/\lambda}, \tag{1}$$

where each $\tilde{a}_i$ is an interval number, and λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$.

It includes a wide range of particular cases, such as the UWA (when $\lambda = 1$), the uncertain weighted geometric average (UWGA) (when $\lambda \to 0$), the uncertain weighted quadratic average (UWQA) (when $\lambda = 2$) and the uncertain weighted harmonic average (UWHA) (when $\lambda = -1$). Note that we will use the λ value that better fits our needs in the specific problem considered. It is very common to use $\lambda = 1$ because it provides the classical weighted average adapted for situations that requires the use of interval numbers. However, we may find a lot of other studies that use other values including the analysis of multiplicative preference relations that is assessed with geometric means (Xu, 2007).

2.3. The probabilistic weighted average

The probabilistic weighted averaging (PWA) operator (Merigó, 2012a) is an aggregation operator that unifies the probability and the weighted average in the same formulation considering the degree of importance that each concept has in the aggregation. It is defined as follows.

Definition 2. A PWA operator of dimension n is a mapping PWA: $R^n \to R$ such that: