European Journal of Operational Research 235 (2014) 247–251

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Computation of the optimal tolls on the traffic network

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ARTICLE INFO

Article history:
Received 18 March 2013
Accepted 19 October 2013
Available online 1 November 2013

Keywords:
Fuzzy optimization
Bilevel optimization
Toll problem

ABSTRACT

The present paper is devoted to the computation of optimal tolls on a traffic network that is described as fuzzy bilevel optimization problem. As a fuzzy bilevel optimization problem we consider bilinear optimization problem with crisp upper level and fuzzy lower level. An effective algorithm for computation optimal tolls for the upper level decision-maker is developed under assumption that the lower level decision-maker chooses the optimal solution as well. The algorithm is based on the membership function approach. This algorithm provides us with a global optimal solution of the fuzzy bilevel optimization problem.

1. Introduction

We discuss hierarchical problems of two decision makers, in which one – the so-called leader – has the first choice and the other one – the so-called follower – reacts optimally on the leader's selection. It is important to note, that each decision-maker maximizes his/her own benefits independently, but is affected by actions of the other decision-maker (through externalities). The formulation of the bilevel programming problem for crisp (i.e. with exactly known and fixed) data can be found in, e.g. (Dempe, 2002).

Since its first formulation by Heinrich von Stackelberg in market economy (in the context of unbalanced economic markets), bilevel optimization has successfully been applied to many real world problems. For the past twenty years transportation problems have been benefiting from the formulation of advances in bilevel programming: (Ben-Ayed, Blair, Boyce, & LeBlanc, 1992; Brotcorne, Labbé, Marcotte, & Savard, 2001; Dempe, Fanghänel, & Starostina, 2009; Kim & Suh, 1988; Labbè, Marcotte, & Savard, 1998; Migdalas, 1995), which cover issues like network design, revenue management and other traffic control problems (where the transportation problem is on the lower level, depending on the parameter selected from the upper level).

Considering the inherently difficult nature of bilevel problems due to their nonconvexity, nonsmoothness and implicitly determined feasible set, it is difficult to design convergent algorithms, and the few algorithms that converge appear to be very slow most of the time. The reason that a global optimal solution of the bilevel optimization problem is difficult to compute is due to $\mathcal{NP}$-hardness of this problem (see Ben-Ayed & Blair, 1990; Blair, 1992).

More often than not we have to make decisions without being able to rely on precise information. Thus, it makes perfect sense to speak about fuzzy optimization problems from a vague predicate approach, as it is understood that this vagueness arises from the way we express the decision-makers' knowledge and not from any random event. These problems proved to be very useful in many applied sciences, such as engineering, economics, applied mathematics, physics, as well as in other disciplines: (Chanas & Kuchta, 1996; Jiménez, Cadenas, Sánchez, Gómez-Skarmeta, & Verdegay, 2006; Peidro, Mula, Jiménez, & Botella, 2010; Shih & Lee, 1999; Weber, Werners, & Zimmerman, 1990; Zimmermann, 1976).

While some convergent algorithms for crisp bilevel problems already exist in the literature (see e.g. Bard, 1982; Bard & Moore, 1990; Bialas & Karwan, 1984; Dempe, 1987; Ishizuka & Aiyoshi, 1992; Önal, 1993; Tuy & Ghannadan, 1998; Wen & Huang, 1996; White & Anandalingam, 1993; Wu, Chen, & Marcotte, 1998), solution strategies for fuzzy bilevel programming problems are an emerging new field with a wide range of practical applicability.

The present work is devoted to the computation of optimal tolls on a traffic network that is described as a fuzzy bilevel optimization problem. As a fuzzy bilevel optimization problem we consider bilinear optimization problem with crisp upper and fuzzy lower levels.

An effective algorithm for computation optimal tolls for the upper level decision-maker is developed under assumption that the lower level decision-maker chooses the optimal solution as well.

The algorithm is based on the membership function approach (Dempe & Ruziyeva, 2012). The lower level fuzzy optimization problem is solved by a method of level-cuts (see Dempe &

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http://dx.doi.org/10.1016/j.ejor.2013.10.059

Ruziyeva, 2011; Ruziyeva & Dempe, 2012 for nonlinear case). Elements of the Pareto set of the corresponding biobjective optimization problem are interpreted as potential optimal solutions of the lower level fuzzy optimization problem on a certain level-cut. The optimal solution is selected due to the highest membership function value.

Then, this solution is used on the upper level such that, with respect to its region of stability, the optimal solution of the leader is found. For computation the regions of stability, Yager ranking indices are used (Liu & Kao, 2004; Ruziyeva & Dempe, 2013). Comparing all optimal solutions with respect to the upper level function value, the global optimal solution is found.

The roadmap of the present paper is the following. In Section 2 the general formulation of the fuzzy bilevel traffic network problem is given.

In Section 3 the solution approach to a fuzzy bilevel optimization problem is described.

The structured algorithm is presented in Section 4.

In Section 5 an illustrative example is solved with the help of the proposed approach.

The conclusions are presented in Section 6.

2. Formulation

Let us assume that the government – as an upper level decision-maker – wants to restrict the traffic flow for the trucks on some streets near a certain landmark (e.g. a school, cultural heritage sites). One possible way is the use of tax-policy on some streets near this landmark. Certainly, the users of the network – the lower level decision-maker – try to minimize the overall costs and choose the cheaper path. But the price does not depend only on the taxes, but on, e.g. fuel, etc.

The mathematical model of this problem is the following. There the function $F(c^{toll},x) = (c^{toll})^T x$ measures the collected money ($c^{toll} \in C^{toll} \subset \mathbb{R}^{|E|}_+$) through a traffic network $G = (V,E)$ consisting of a node set V for junctions and an edge set E containing all streets connecting the junctions.

Assume that the function $\widetilde{f}(c^{toll},x)$ measures the quality of the traffic flow x and depends on the crisp toll charges c^{toll} and the fixed usual user's fuzzy costs such as, e.g. fuel $\tilde{c}$. Possible realization is $\widetilde{f}(c^{toll},x) = (c^{toll} + \tilde{c})^T x$.

Computing the system optimum in the traffic network means that we maximize the collected money for the leader (the government) as the upper level objective function value $F(c^{toll},x)$ and minimize the total costs for the travel of all the users (that we describe as one follower) in the network as the lower level objective function value $\widetilde{f}(c^{toll},x)$. Clearly these costs do not only depend on the fuzzy costs $\tilde{c}_e$ for traversing the edge $e \in E$ but also on the collected toll charges c^{toll}_e. Let $C^{toll} := \{c^{toll}_e : e \in E\}$ and assume that $c^{toll}_e \geqslant 0$. If for some $e_0 \in E$ $c^{toll}_{e_0} = 0$ we say, that e_0 is a *toll-free* path.

Using this problem a user equilibrium traffic flow can be computed provided that the costs $\tilde{c}_e$ are approximately known, i.e. have fuzzy values, and depend on the flow over this edge. Using fuzzy travel costs, the computation of the traffic flow reduces to a fuzzy network flow problem.

Assume that v units of a certain commodity should be transported with minimal for the users (follower) overall costs from the origin $s \in V$ to the destination $d \in V$ through the traffic network G. The problem on the lower level is to compute optimal amounts of transported commodities on the streets of the network.

Let $x_e = x_{kl}$ denote the amount of transported units over the edge $e = (k,l) \in E$, that connects two vertices k and l $(k,l \in V)$. Let O_k (I_k) denote the set of all edges leaving (entering) the node k.

Assume that the streets may have different capacities and let the flow x_e on the edge e be bounded by the capacity u_e. This is expressed in the inequality (3) given below. The constraint (4) is used to guarantee that the total incoming flow is equal to the total outgoing flow. Moreover, the outgoing flow in the origin equals to v (see (5)).

In given terms it is formulated as follows.

$$F(c^{toll},x) = \sum_{e\in T} c^{toll}_e x_e \to \max_{c^{toll}_e \in C^{toll}} \tag{1}$$

subject to x solving

$$\widetilde{f}(c^{toll},x) = \sum_{e\in E}\sum_{e\in T} c^{toll}_e x_e + \tilde{c}_e x_e \to \min_x \tag{2}$$

$$x_e \le u_e \qquad \forall e \in E \tag{3}$$

$$\sum_{k\in I_l} x_{kl} - \sum_{i\in O_l} x_{li} = 0, \quad \forall\, l \in V \setminus \{s,d\} \tag{4}$$

$$\sum_{k\in I_s} x_{ks} - \sum_{i\in O_s} x_{si} = -v, \tag{5}$$

$$x_e \ge 0 \tag{6}$$

3. Solution method

It is easy to see, that (1)–(6) is a fuzzy bilevel optimization problem. Let us pose (1)–(6) in a compact form of

$$\begin{aligned} &F(c^{toll},x) \to \min_{c^{toll} \in C^{toll}} \\ &\text{s.t. } \widetilde{f}(c^{toll},x) \to \min_{x\in X}, \end{aligned} \tag{7}$$

where polytope X is a set of solutions of (3)–(6)

$$\Psi(c^{toll}) = \arg\min_x \left\{ \widetilde{f}(c^{toll},x) : x \in X \right\}$$

is the optimal solution set of lower level problem (2)–(6).

There exist only few possibilities to deal with bilevel optimization problem. Namely,

1. Assume that the follower always selects the optimal decisions which give the worst values of $F(c^{toll},x)$. This is the *pessimistic* or *strong approach* (Dempe, 2002), which is used when the leader is not able to influence the follower and is forced to choose an approach bounding the damage resulting from an unfavorable selection by the follower. The resulting problem is:
$$\min_{c^{toll}\in C^{toll}} \phi_p(c^{toll}),$$
where $\phi_p(c^{toll}) = \max_{x\in\Psi(c^{toll})} F(c^{toll},x)$.
2. Assume that the leader is able to influence the follower, so that the last one always selects the variables x to provide the best value of $F(c^{toll},x)$. This results in the so-called *optimistic* or *weak approach* (Dempe & Starostina, 2007), where the resulting problem is stated as
$$\min_{c^{toll}\in C^{toll}} \phi_o(c^{toll}),$$
where $\phi_o(c^{toll}) = \min_{x\in\Psi(c^{toll})} F(c^{toll},x)$.
3. In the case when none of the assumptions described earlier can be conceded, selection function approach (Dempe & Starostina, 2006) can be applied to the initial fuzzy bilevel optimization problem.

The selection approach 3. is quite new and it seems to be more appropriate for our needs. Let us denote some element of $\Psi(c^{toll})$ by $x(c^{toll})$ and assume, that this choice is a fixed selection function for all possible $c^{toll} \in C^{toll}$ (see Dempe & Starostina, 2006). The vector of parameters c^{toll} describes the "environmental data" for the fuzzy lower level problem.