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Continuous Optimization

Global optimization of signomial geometric programming problems

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ABSTRACT

This paper presents a global optimization approach for solving signomial geometric programming problems. In most cases nonconvex optimization problems with signomial parts are difficult, NP-hard problems to solve for global optimality. But some transformation and convexification strategies can be used to convert the original signomial geometric programming problem into a series of standard geometric programming problems that can be solved to reach a global solution. The tractability and effectiveness of the proposed successive convexification framework is demonstrated by seven numerical experiments. Some considerations are also presented to investigate the convergence properties of the algorithm and to give a performance comparison of our proposed approach and the current methods in terms of both computational efficiency and solution quality.

1. Introduction

Optimization problems that contain signomial expressions in the objective and constraint functions are usually called Signomial Geometric Programming (SGP) problems. This kind of optimization problems have a wide range of applications in engineering, science, management, etc. Examples of application of SGP are engineering design (Avriel & Barrett, 1978; Dembo & Avriel, 1978; Maranas & Floudas, 1997; Marín-Sanguino, Voit, González-Alcón, & Torres, 2007; Xu, 2013), inventory control (Jung & Klein, 2005; Kim & Lee, 1998; Mandal, Roy, & Maiti, 2006), project management (Scott & Jefferson, 1995), power control (Chiang, Tan, Palomar, O'Neill, & Julian, 2007), etc. Some comprehensive surveys of these applications can be found in Maranas and Floudas (1997), Floudas (2000), Biegler and Grossmann (2004), Chiang (2005), Boyd, Kim, Vandenberghe, and Hassibi (2007), Floudas and Gounaris (2009), Lin, Tsai, and Yu (2012).

Since the SGP problems belong to a truly nonconvex class of problems that is an intrinsically intractable NP-hard problem, these problems are difficult to solve for global optimality. In the last decades, some research has been directed toward the development of global optimization strategies for SGP problems (Chiang et al., 2007; Floudas, 2000; Lange & Zhou, in press; Li & Lu, 2009; Lin & Tsai, 2012; Lundell & Westerlund, 2009a, 2009b; Lundell, Westerlund, & Westerlund, 2009; Maranas & Floudas, 1997; Pörn, Björk, & Westerlund, 2008; Qu, Zhang, & Ji, 2007; Shen, 2005; Toscano & Amouri, 2012; Tsai & Lin, 2008, 2011; Tsai, Lin, & Hu, 2007; Wang, Zhang, & Gao, 2004; Westerlund, 2007). For example, Maranas and Floudas (1997), Floudas (2000) proposed a branch-and-bound based global optimization method for the solution of SGP problems by using the exponential variable transformation and convex underestimation. Shen (2005) globally solved SGP problems through a series of linear programming problems. Chiang et al. (2007) presented a heuristic strategy that can compute truly nonconvex power control problems by solving a sequence of geometric programming (GP) problems through the method of successive convex approximations. In the practical implementation of this approach, however, more iterations are possibly required to find the approximately global optimum of a SGP problem (see Section 3.2, Example 6). Li and Lu (2009) developed a method to deal with SGP problems with mixed free-sign variables using some linearization and convexification techniques. Lundell et al. (2009) presented some power transformation and piecewise linear approximation strategies to reach a global optimal solution of optimization problems including signomial terms. Toscano and Amouri (2012) introduced some simple approaches for easily solving a kind of nonconvex problems, called quasi geometric programming problems. Lange and Zhou (in press) applied the geometric arithmetic mean inequality and a supporting hyperplane inequality to derive an MM algorithm for SGP problems. Tsai and Lin (2011) proposed an approach for solving a posynomial geometric programming with separable functions by utilizing superior piecewise linear functions and efficient variable transformations. This method requires much time to reach an approximate global solution of SGP problems (Lin & Tsai, 2012). To handle this difficulty of computational burden, Lin and Tsai (2012) improved the Tsai and Lin (2011) approach by using the range reduction strategies to decrease the CPU time in treating the SGP problems. Two possible outcomes produce when this modified version is used to solve a SGP problem. One is that the Lin and Tsai (2012) method requires much time to reach the globally optimal solutions of some SGP problems (see Examples 2–4 in Lin & Tsai (2012)). The other is that the solution

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obtained by the Lin and Tsai (2012) method is not a global optimum but a local one of CSTR sequence design problem in Lin and Tsai (2012) (see also Section 3.2, Example 5).

In this paper, we present an iterative method to efficiently find the globally optimal solution of an SGP problem. The approach proposed relies on posing the nonconvex SGP problem as a standard GP problem by using simple transformation and condense techniques. The resulting optimization problem can be solved very efficiently by a sequence of standard GPs. We demonstrate the capabilities of the proposed algorithm through seven numerical examples, comparing our results with those produced by other methods. Numerical experiments show that the proposed algorithm requires much less CPU time to obtain the global optimum of a SGP problem with lower errors in objective and constraint functions than the current approaches do.

The rest of this paper is organized as follows. Section 2 describes the SGP problems and presents the global optimization method for solving SGP problems. Then seven numerical examples taken from the literature are presented to illustrate the tractability and effectiveness of the proposed approach in computational efficiency and solution quality. Finally, brief conclusions are given in Section 4.

2. Global optimization of signomial geometric programming problems

2.1. Signomial geometric programming problems

Let us consider a signomial geometric programming (SGP) in the following form:

$$\min \quad f_0(x) = \sum_{j=1}^{m_0} c_{0j} \prod_{i=1}^{n} x_i^{a_{0ij}} \tag{1}$$

subject to satisfying:

$$f_k(x) = \sum_{j=1}^{m_k} c_{kj} \prod_{i=1}^{n} x_i^{a_{kij}} \leqslant 1, \quad k = 1, 2, \ldots, p \tag{2}$$

$$f_k(x) = \sum_{j=1}^{m_k} c_{kj} \prod_{i=1}^{n} x_i^{a_{kij}} = 1, \quad k = p+1, p+2, \ldots, q \tag{3}$$

$$x_i > 0, \quad i = 1, 2, \ldots, n \tag{4}$$

where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in \mathrm{R}^n$, $c_{kj} \in \mathrm{R}$, $a_{kij} \in \mathrm{R}$, m_k $(k = 0, 1, \ldots, q)$ represents the number of product terms of the objective function and of the constraints, and $\prod_1^n x_i^{a_{kij}}$ and f_k $(k = 0,\ 1, \ldots, q)$ are monomial and signomial functions respectively.

The SGP problem as shown in Eqs. (1)–(4) is a highly nonlinear, nonconvex optimization problem and difficult to solve for global optimality.

2.2. Global optimization method

This paper proposes a global optimization approach for solving SGP problems. Some transformation and convexification strategies are applied to convert the original SGP problem into a sequence of standard geometric programming (GP) problems that can be solved to reach a global solution.

We first denote all functions f_k $(k = 0, 1, \ldots, q)$ in Eqs. (1)–(3) as

$$f_k = f_k^+(x) - f_k^-(x), \quad k = 0, 1, \ldots, q \tag{5}$$

where $f_k^+(x)$ and $f_k^-(x)$ have the following posynomial formulations:

$$f_k^+(x) = \sum_{j \in J_k^+} c_{kj} \prod_{i=1}^{n} x_i^{a_{kij}}, \quad k = 0, 1, \ldots, q$$

$$f_k^-(x) = \sum_{j \in J_k^-} (-c_{kj}) \prod_{i=1}^{n} x_i^{a_{kij}}, \quad k = 0, 1, \ldots, q$$

where $J_k^+ = \{j | j \in J_k, c_{kj} > 0\}$ and $J_k^- = \{j | j \in J_k, c_{kj} < 0\}$ with $J_k = \{1, 2, \ldots, m_k\}$.

Then optimization problem (1)–(4) can be written as the following equivalent formulation:

$$\min \quad f_0^+(x) - f_0^-(x) + M \tag{6}$$

subject to satisfying:

$$f_k^+(x) - f_k^-(x) \leqslant 1, \quad k = 1, 2, \ldots, p \tag{7}$$

$$f_k^+(x) - f_k^-(x) = 1, \quad k = p+1, p+2, \ldots, q \tag{8}$$

$$x_i > 0, \quad i = 1, 2, \ldots, n \tag{9}$$

where $M > 0$ is a sufficiently large constant. The reason for using $f_0^+(x) - f_0^-(x) + M$ instead of $f_0^+(x) - f_0^-(x)$ to be the objective function is that a sufficiently large M value will force $f_0^+(x) - f_0^-(x) + M > 0$.

Next we introduce an additional variable x_0 to create a linear objective function and rearrange the constraints into quotient form to obtain the following equivalent optimization problem:

$$\min \quad x_0 \tag{10}$$

subject to satisfying:

$$\frac{f_0^+(x) + M}{f_0^-(x) + x_0} \leqslant 1 \tag{11}$$

$$\frac{f_k^+(x)}{f_k^-(x) + 1} \leqslant 1, \quad k = 1, 2, \ldots, p \tag{12}$$

$$\frac{f_k^+(x)}{f_k^-(x) + 1} = 1, \quad k = p+1, p+2, \ldots, q \tag{13}$$

$$x_i > 0, \quad i = 0, 1, \ldots, n \tag{14}$$

In this representation, constraints (11)–(13) involve a special structure in the form of a ratio between two posynomials. This kind of optimization problems as shown in Eqs. (10)–(14) belong to a truly nonconvex class of problems known as complementary geometric programming (CGP) (Chiang, 2005; Chiang et al., 2007) that is an intrinsically intractable NP-hard problem.

Now we introduce auxiliary variables t_k and rewrite optimization problem (10)–(14) as

$$\min \quad x_0 + \sum_{k \in K_{22} \cup K_{23} \cup K_{24}} w_k t_k \tag{15}$$

subject to satisfying:

$$\frac{f_0^+(x) + M}{f_0^-(x) + x_0} \leqslant 1 \tag{16}$$

$$\frac{f_k^+(x)}{f_k^-(x) + 1} \leqslant 1, \quad k \in K_{11} \tag{17}$$

$$\frac{f_k^+(x)}{f_k^-(x) + 1} \leqslant 1, \quad k \in K_{12} \tag{18}$$

$$\frac{f_k^+(x)}{f_k^-(x) + 1} = 1, \quad k \in K_{21} \tag{19}$$

$$\frac{f_k^+(x)}{f_k^-(x) + 1} \leqslant 1, \quad k \in K_{22} \tag{20}$$

$$\frac{f_k^+(x)}{f_k^-(x) + 1} \geqslant 1 - t_k, \quad k \in K_{22} \tag{21}$$

$$\frac{f_k^-(x) + 1}{f_k^+(x)} \leqslant 1, \quad k \in K_{23} \tag{22}$$

$$\frac{f_k^-(x) + 1}{f_k^+(x)} \geqslant 1 - t_k, \quad k \in K_{23} \tag{23}$$

$$\frac{f_k^+(x)}{f_k^-(x) + 1} \leqslant 1, \quad k \in K_{24} \tag{24}$$

$$\frac{f_k^+(x)}{f_k^-(x) + 1} \geqslant 1 - t_k, \quad k \in K_{24} \tag{25}$$

$$x_i > 0, \quad i = 0, 1, \ldots, n \tag{26}$$

$$0 \leqslant t_k \leqslant 1, \quad k \in K_{22} \cup K_{23} \cup K_{24} \tag{27}$$