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A compound control chart for monitoring and controlling high quality processes

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ABSTRACT

In the present article, we propose a new control chart for monitoring high quality processes. More specifically, we suggest declaring the monitored process out of control, by exploiting a compound rule couching on the number of conforming units observed between the $(i-1)$th and the ith nonconforming item and the number of conforming items observed between the $(i-2)$th and the ith nonconforming item. Our numerical experimentation demonstrates that the proposed control chart, in most of the cases, exhibits a better (or at least equivalent) performance than its competitors.

1. Introduction

Attribute data arise regularly in the context of Statistical Process Control (see for example Wu, Zhang, & Yeo (2001), Liu, He, Shu, & Wu (2009), Quinino, Colin, & Ho (2010), and Haridy, Wu, Lee, & Bhuiyan (2013)). A typical example of data of this type arises when a product is classified as conforming or nonconforming. In such cases, the most popular chart used by practitioners for detecting whether the fraction of defective (nonconforming) products has shifted away from its nominal value is a *p* or an *np* chart (see for example De Araújo Rodrigues, Epprecht, & De Magalhães (2011)). For an extensive review of research on control charts for attribute data the interested reader is referred to Woodall (1997) while excellent introductory material on this topic may be found in the monographs of Duncan (1986) and Montgomery (2005).

The continuous effort on improving the quality of manufactured products has offered efficient tools to set up processes with very small number of non-conforming items (Xie & Goh, 1992, 1993). Such processes are typically called *high quality processes* (or zero-defects processes). Paradoxically, although the *p* and *np* charts have been proven quite effective in the course of time, they are incapable of monitoring high quality processes (see for example Wang (2009)). This is mainly due to the fact that the rules used in these charts are couched on the fraction of non-conforming items (or defects) appearing in the sample under inspection; hence, if a small or moderate shift occurs in a zero-defect process, the out-of-control fraction of non-conforming items will still be very small, and as a consequence, it is highly probable, that no defective items will be observed in the inspected sample. Therefore, for small or moderate shifts, standard *p* or *np* charts fail to diagnose a change in a high quality process.

For this reason, in case of processes involving small fractions of non-conforming units, the use of standard *p* or *np* charts may require that the practitioner enlarge dramatically the sample size or consider up to 100% inspection (Bourke, 1991).

Another practical drawback of the standard *p* or *np* charts is the intuitively meaningless lower control limit, which in most of the cases takes on a negative value and it is therefore set equal to zero; in this case the chart does not have the potential to detect process improvement.

The aforementioned shortcomes of the standard Shewhart *p* or *np* charts, when applied in high quality processes, motivated extensive research interest towards establishing alternative controlling schemes that avoid the use of the number of non-conforming items in the sample under inspection. We recall that, when very small fractions of non-conforming items (small values of *p*) are observed, the number of defective items can be approximated by an appropriate Poisson distribution, while the number of non-defective items between these defective items may be described by an appropriate exponential distribution. Charts based

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on such approximations have been introduced and studied by Nelson (1994).

A similar idea, initially launched by Calvin (1983) and further studied by Goh (1987a, 1987b) is to use the exact distribution of the number of non-defective items between consecutive defective items. This approach gave birth to a new control chart, which exploits the cumulative count of conforming items between two successive non-conforming items in order to construct the rule that initiates the out of control signal; the name used for this chart is CCC_1 (Cumulative Count Conforming) control chart.

A direct modification of CCC_1 chart, is the so-called CCC_r chart which exploits the number of inspected items until r nonconforming items are observed (Bourke, 1991; Ohta, Kusukawa, & Rahim, 2001; Chan, Lai, Xie, & Goh, 2003). The CCC_r chart is more sensitive to small upward shifts as r increases. However, when large values of r are used, CCC_r becomes very ineffective in detecting large upward shifts and requires a larger number of items to be tracked down, a fact that results in a substantial cost upsurge.

Tang and Cheong (2006) introduced a modified Cumulative Count Conforming chart for monitoring high quality processes when inspection is carried out in groups while Niaki and Abbasi (2007) dealt with the problem of monitoring multi attributed high quality processes. Albers (2010) studied the problem of optimal design of negative binomial charts, Chen, Chen, and Chiou (2011) presented a *CCC* chart with variable sampling intervals and control limits while Albers (2011) introduced a class of nonparametric control charts for high quality processes. All the charts mentioned so far require 100% inspection. For an approach allowing for periods when no inspection is exercised see Reynolds and Stoubos (1999).

Besides the Shewhart-type control charts reviewed above, CUSUM and EWMA control charts have also been suggested in the quality control literature for monitoring high quality processes, see Yeh, Mcgrath, Sembower, and Shen (2008) and Szarka and Woodall (2012). However, in most of the cases they are too complicated for the practitioners.

A reasonable question arising from the foregone discussion is how one could monitor the stability of a high quality process, by exploiting simple to apply detection schemes that will work efficiently for both moderate and large shifts.

Motivated by this observation, in the present article, we propose a control chart that uses a simple to apply compound rule for monitoring high quality processes. In Section 2, we review the CCC_r control charts which are the basic ingredient of our compound rule. In Section 3, we provide the necessary notations and introduce the new control chart while in Section 4, we suggest a technique for assessing the run length distribution of the new control chart by exploiting a Markov Chain imbedding method. A comparative study of the performance of the suggested monitoring scheme against other competing techniques is given in Section 5. Finally, in Section 6 we present two real data examples followed by some conclusions.

2. The CCC_r control charts

As already mentioned, in the case of high quality processes it is difficult to identify even large shifts since only a few defects occur. To address this problem, it seems plausible to couch our decision on the cumulative count of conforming items (CCC_1 rule) between two consecutive nonconforming ones. Apparently, small values of the cumulative count of conforming items correspond to increased number of nonconforming products and this is an indication of a possible shift in the process. On the contrary, if large values of cumulative count of conforming items are observed one may conclude that the process is in control.

Let Y_1 denote the number of conforming units till the appearance of the first nonconforming unit and Y_i, $i = 2, 3, \ldots$ the number of conforming units between the $(i-1)$th and the ith nonconforming items. Apparently, if we assume that consecutive products are serially independent (this is the typical case in most process control applications) and denote by p the fraction of nonconforming items of the monitored process, then the random variable $Y_i + 1$ (number of items inspected after the occurrence of the $(i-1)$th nonconforming item to the ith nonconforming item) will follow a standard geometric distribution with probability mass function

$$P(Y_i + 1 = n) = p(1-p)^{n-1}, \quad n = 1, 2, \ldots.$$

When a false alarm level at most a is set, the exact probability limits LCL, UCL for CCC_1 should satisfy the inequalities

$$UCL \geqslant \frac{\ln(a/2)}{\ln(1-p)}, \quad LCL \leqslant \frac{\ln(1-a/2)}{\ln(1-p)}$$

It is worth mentioning that, in most publications, these formulae are given in the form of equalities; however if we wish LCL, UCL to be positive integers, we should identify them by the aid of the aforementioned inequalities. The use of exact probability limits is quite popular in the quality control literature, see e.g. Calvin (1983) and Wetherill and Brown (1991). For a discussion of control charts associated to the geometric distribution see Kaminsky, Benneyan, Davis, and Burke (1992).

In a CCC_1 chart, the observed values of $Y_i + 1$, $i = 1, 2, 3, \ldots$, are plotted against i and an out of control signal is issued at time $\min\{i: Y_i + 1 \geqslant UCL \text{ or } Y_i + 1 \leqslant LCL\}$. If a plotted point falls below LCL it seems reasonable to conclude that the process has deteriorated (i.e. an increase in the value of p has occurred) while should a point be plotted above UCL it may be inferred that a possible improvement in the process has occurred.

A straightforward generalization of the CCC_1 chart principle arises if, instead of looking at the cumulative count of conforming items between two consecutive nonconforming units, we couch our decision on the total number of conforming items between $r \geqslant 1$ consecutive nonconforming units. The resulting control chart will be called CCC_r chart.

In order to describe the underlying probability model in a CCC_r chart, let us denote by $Y_{r,i} = \sum_{j=0}^{r-1} Y_{i-j}, i = r, r+1, \ldots$ the number of conforming units between the $(i-r)$th and ith nonconforming items. If p stands for the fraction of nonconforming items in an in-control process, then $Y_{r,i} + r$, $i = r, r+1, \ldots$ will follow a standard negative binomial distribution with probability mass function

$$P(Y_{r,i} + r = n) = \binom{n-1}{r-1} p^r (1-p)^{n-r}, \quad n = r, r+1, \ldots.$$

The lower and upper control limits LCL, UCL should satisfy the following inequalities

$$\sum_{i=UCL}^{\infty} \binom{i-1}{r-1} p^r (1-p)^{i-r} \leqslant a/2, \quad \sum_{i=r}^{LCL} \binom{i-1}{r-1} p^r (1-p)^{i-r} \leqslant a/2,$$

where α is a prespecified (maximum) level of false alarm. In the resulting CCC_r chart plot, the observed values of $Y_{r,i} + r$ should be plotted against $i = r, r+1, r+2, \ldots$ and a signal will be issued at time $\min\{i: Y_i + r \geqslant UCL \text{ or } Y_i + r \leqslant LCL\}$.

Since the use of large r values will result in a rapid upsurge of the lower control limit as the process fraction nonconforming p approaches zero, it is advisable that small values of r be used when a high quality process is to be monitored.