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Short Communication

A note on supply chain coordination for joint determination of order quantity and reorder point using a credit option

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ABSTRACT

Credit options and side payments are two methods suggested for achieving coordination in a two-echelon supply chain. We examine the credit option coordination mechanism introduced by Chaharsooghi and Heydari [Chaharsooghi, S., & Heydari, J. (2010). Supply chain coordination for the joint determination of order quantity and reorder point using credit option. *European Journal of Operational Research, 204*(1), 86–95]. This method assumes that the supplier's opportunity costs are equal to the reduction in the buyer's financial holding costs during the credit period. In this note, we show that Chaharsooghi and Heydari's method is not applicable when buyer and supplier opportunity costs are not equal. We introduce an alternate per order rebate method that reduces supply chain costs to centralized management levels.

1. Introduction

This note is concerned with a two-echelon supply chain (SC) consisting of a buyer and a supplier. The buyer seeks to fulfill the randomly received orders of its customers while minimizing its inventory holding, ordering, and shortage costs. Many previous studies have proposed models that provide mechanisms for the parties in a two-echelon SC to coordinate their inventory policies. These incentives include quantity discounts, side payments, buyback contracts, and credit options. Li and Wang (2007) published a review of research in this area in the *European Journal of Operational Research*.

This note addresses the case where the buyer in a two-echelon SC selects an order quantity and a reorder point as policies in a continuous review model. There are a limited number of previous studies that specifically address inventory management in a SC under these assumptions where demand is stochastic. Li and Liu (2006) find analytical solutions for an optimal order quantity given a previously established reorder point when a quantity discount is used to coordinate the SC. Shin and Benton (2007) consider a situation where the buyer uses its economic order quantity but selects a reorder point that minimizes inventory and transportation costs. Chaharsooghi, Heydari, and Kamalabadi (2011) expand the Li and Liu (2006) quantity discount model to a setting where both order quantity and reorder point can be selected to minimize SC costs.

Credit options and per order rebate payments as coordinating incentives in SCs where the buyer operates under a continuous review inventory system will be examined. Demand is a random variable and both the inventory order quantity and a safety stock level will be chosen with the goal of minimizing inventory costs in the SC. A method suggested by Chaharsooghi and Heydari (2010) (hereafter CH) that selects an appropriate credit term to coordinate inventory policies will be evaluated.

To employ the CH technique, the supplier allows the buyer to delay payment for a portion of each order cycle, in exchange for agreeing to establish inventory policies that would be optimal if the SC were centralized. The benefit for the buyer is a reduction in the financial portion of its holding costs during the credit period. The supplier incurs opportunity costs associated with the delay in receiving its payment for the delivered goods.

The CH method assumes the reduction in holding costs for the buyer are exactly equal to the credit incentive costs paid by the supplier. This assumes that the supplier's opportunity costs of delaying receipt of its payment is somehow determined by the buyer's inventory level and the buyer's financial holding cost. A modified inventory cost function for the supplier that considers its true opportunity costs is presented. The range of potential values for the credit term where both parties benefit is narrower than shown by CH and the method is not guaranteed to reduce SC costs. After a study of the CH model, an alternate SC coordination plan is

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proposed that employs a per order rebate as an incentive for participation.

The remainder of the note is organized as follows. Sections 2 and 3 define notation and describe the CH credit option model. Section 4 illustrates the application of the credit option model through analysis of an example problem. Section 5 introduces an alternate method that employs a per order rebate to the buyer as a coordinating incentive. Section 6 presents results from three test problems. Section 7 summarizes and concludes the paper.

2. Notation

In the two-echelon SC considered in this note, the buyer randomly receives demand for a product from its customers and in turn places orders for good from the supplier. The supplier receives these orders and ships inventory to the buyer.

A significant portion of the notation from CH has been retained, but since the holding costs in the model will be explicitly divided into storage and financial holding costs, some terminology is slightly different. The notation used for the model is shown in Table 1. Parameters for test problems used by CH are also shown in this table and will be referenced later.

Subscripts b and s will be used to represent results for the buyer and supplier, respectively. Subscripts or superscripts d and c denote results obtained from operating in the decentralized and centralized modes, respectively.

The model under consideration allows a buyer experiencing uncertain demand to select both an order quantity and safety stock factor to minimize its costs. Both decentralized and centralized modes of operation will be considered. In the decentralized mode, the buyer selects optimal values for the decision variables that minimize its own costs without regard for the supplier's costs. The supplier reacts by selecting its own order quantity based on the number of units ordered by the buyer. In the centralized mode, the buyer and supplier simultaneously select their order quantities and safety stock level to minimize the total SC costs. If the SC is decentralized, a coordination mechanism may entice the buyer to choose the optimal centralized order quantity and safety stock level in exchange for an incentive (a credit option and a per order rebate will be considered).

Table 1
Notation and parameters for the CH test problems.

Value	Description	Test problem values
Q	Buyer's order quantity each replenishment cycle	Decision variable
k	Buyer's safety stock factor	Decision variable
N	Number of buyer replenishment cycles covered by a supplier order (integer)	Decision variable
C	Credit time period offered to buyer by supplier (in years)	Decision variable
V	Rebate per order offered to buyer by supplier	Decision variable
D	Expected annual demand	500, 700, 2000
h_b	Buyer's annual financial holding cost percentage	0.028, 0.06, 0.04
s_b	Buyer's storage cost per unit per year	1.4, 12, 1
P_1	Inventory purchase price of buyer from supplier	200, 50, 100
h_s	Supplier's annual financial holding cost percentage	0.03, 0.2, 0.1
s_s	Supplier's storage cost per unit per year	2.5, 1, 7.5
P_2	Inventory purchase price of supplier from external vendor	150, 20, 75
A_b	Buyer's ordering cost per replenishment	100, 100, 50
A_s	Supplier's setup cost per order	100, 300, 150
B	Shortage cost per unit short	12, 15, 6
σ_X	Standard deviation of demand during lead time	20, 100, 50
μ_X	Expected demand during lead time	

3. Coordinating a supply chain with a credit period

The model defined in this section is similar to the one defined by CH, except that the problem has been re-stated to explicitly show the financial and storage parts of the holding costs of the buyer and supplier as separate values.

The expected annual cost function for the buyer is

$$TC_b(Q,k,C) = \left[A_b \cdot \frac{D}{Q}\right] + \left[B \cdot S_k(k) \cdot \sigma_X \cdot \frac{D}{Q}\right] + (s_b + P_1 \cdot h_b) \cdot [0.5Q + k \cdot \sigma_X] - CI(Q,k,C), \quad (1)$$

where the coordination incentive, CI, is

$$CI(Q,k,C) = \frac{P_1 \cdot h_b \cdot D}{Q} \cdot C(Q - 0.5DC + k\sigma_X). \quad (2)$$

The coordination incentive is equal to the financial holding costs of the buyer during the credit period of length C (measured in years); this is equal to the financial holding cost per unit per year, $P_1 \cdot h_b$, multiplied times the area from time [0, C] in Fig. 1 for each order cycle. The terms preceding CI in (1) are the ordering cost, shortage cost, and holding cost before considering savings due to credit terms, all measured on an annual basis.

In the buyer's annual expected cost Eq. (1), the firm is assumed to hold average safety stock of $k \cdot \sigma_X$. This is the product of the safety stock factor and the standard deviation of lead time demand, which is assumed to be normally distributed. The buyer's inventory reorder point is determined as $\mu_X + k \cdot \sigma_X$. The expected shortage factor per replenishment cycle, S_k, is calculated for a given k as

$$S_k(k) = \int_k^\infty (z-k) \cdot \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz. \quad (3)$$

The supplier receives orders of size Q from the buyer. CH assume the supplier's order quantity will always be an integer multiple (N, $N \geqslant 1$) of Q when costs are minimized, such that the average inventory is $(N-1)Q/2$. This means that the supplier's total cost function is

$$TC_s(Q,k,N,C) = \left[A_s \cdot \frac{D}{NQ}\right] + \left[(s_s + P_2 \cdot h_s) \cdot \frac{(N-1) \cdot Q}{2}\right] + CI(Q,k,C). \quad (4)$$

The supplier's cost function is the sum of its ordering costs, holding costs, and the concession made to the buyer to participate in the coordinated model. The coordination incentive, CI, defined in (2) and deducted from the buyer's cost function is added to the supplier's cost function.

The next three sub-sections describe the optimal inventory policies in the SC under decentralization, centralization, and coordination.

3.1. Decentralized model

In the decentralized mode, the supplier does not offer a credit period to the buyer, so the buyer assumes $C = 0$ and sets the optimal policy by finding

$$(Q_d^*, k_d^*) = \underset{(Q,k)}{\text{ArgMin}}\ \ TC_b(Q,k,0). \quad (5)$$

The resulting total cost is $TC_b^d = TC_b(Q_d^*, k_d^*, 0)$. CH provide a partial solution that states the optimal value for Q as a function of the optimal value of k as

$$Q_d^* = \sqrt{\frac{2D(A_b + BS_k(k_d^*)\sigma_X)}{s_b + P_1 \cdot h_b}} \quad \text{and} \quad 1 - \Phi(k_d^*) = \frac{(s_b + P_1 \cdot h_b)Q_d^*}{DB},$$