European Journal of Operational Research 231 (2013) 598–616

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

# Multi-level lot sizing and job shop scheduling with compressible process times: A cutting plane approach

M. Karimi-Nasab *, S.M. Seyedhoseini

*Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*

ARTICLE INFO

*Article history:*
Received 28 October 2012
Accepted 12 June 2013
Available online 21 June 2013

*Keywords:*
(P) Production
Scheduling
Process compressibility
Job shop
Cutting planes
Multi-level lot sizing

ABSTRACT

This paper proposes an integer linear programming formulation for a simultaneous lot sizing and scheduling problem in a job shop environment. Among others, one of our realistic assumptions is dealing with flexible machines which enable the production manager to change their working speeds. Then, a number of valid inequalities are developed based on problem structures. As the valid inequalities can help in reducing the non-optimal parts of the solution space, they are dealt with as some cutting planes. The proposed cutting planes are used to solve the problem in (i) cut-and-branch, and (ii) branch-and-cut approaches. The performance of each cutting plane is investigated with CPLEX 12.2 on a set of randomly-generated test data. Then, some performance criteria are identified and the proposed cutting planes are ranked by TOPSIS method.



## 1. Introduction

Production management is a multi-disciplinary job that involves in considering many factors simultaneously. Among other tasks, a production manager should simultaneously decide about lot sizing and process routing of some items in each planning period to minimize total cost of the plant. Furthermore, there is limited amount of available working time due to technical limitations in a planning period. And, dealing with such technical limitations results in a complicated problem that was mainly treated by heuristics and rule of thumbs (Fallah & Shayan, 2002). Also, these decisions (i.e. lot sizing and machine sequencing) were traditionally made in a sequential manner, resulting in a number of recursive corrective actions. In more details, the following steps were generally taken in traditional (i.e. MRP-based) production management with some minor variations (Koh, Saad, & Jones, 2002; Pochet & Wolsey, 2006; Waters, 2003):

*Step 0*: estimate the available capacity for processing a set of items over needed machines in the following periods,
*Step 1*: determine lot sizes of different items for forthcoming planning periods,
*Step 2*: determine the best schedule of machineries to perform different processes of each item, regarding their lot sizes,
*Step 3*: check if the obtained production schedule is feasible in term of needed capacities;
  if the production schedule is not feasible, then
  revise the decisions by some heuristics,
  otherwise
    perform a local search to find better solutions for a number of trials.

Moreover, these decisions are usually updated via a rolling horizon framework with the hope of cost reduction by means of special-purpose packages or heuristics (Karimi, Fatemi Ghomi, & Wilson, 2003). Consequently, there are many critiques on the non-optimality of the above procedure due to non-simultaneousness of decision makings (Franck, Neumann, & Schwindt, 1997). On the other hand, machines can work in different rates/modes by changing their settings. This feature is ordinarily observed in lots of real world shop floors and provides a degree of flexibility for a production manager, but it may also complicate determining the optimal policy. So, if we decide about lot sizes and process sequences simultaneously while considering shop constraints, then we can be sure about the efficiency and/or optimality of the whole production schedule (i.e. lot sizes and machine sequences in periods). However, this may lead to mixed integer programming (MIP) formulations that are hard to solve in reasonable time for sizes close to real world (Pochet & Wolsey, 2006). This paper tries to fill in the gap by providing a problem formulation along with a set of efficient valid inequalities.

* Corresponding author. Tel.: +98 9127990527.
*E-mail addresses:* mehdikariminasab@iust.ac.ir (M. Karimi-Nasab), seyedhosseini@iust.ac.ir (S.M. Seyedhoseini).

0377-2217/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2013.06.021

**Table 1**
A summary of the literature on production scheduling.

| Authors (year) | No. products | | No. levels | | Setup | | Planning horizon | | Time consideration | | Objective(s) | | | | Solution method |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Single | Multiple | Single | Multiple | Seq.-dep. | Seq.-indep. | Finite | Infinite | Discrete | Continuous | Makespan | Due-related | Cost | Others | |
| De Bodt et al. (1984) | | | | | | | | | Literature review | | | | | | |
| Afentakis (1985) | | ✓ | | ✓ | | ✓ | ✓ | | ✓ | | | | ✓ | | – |
| Ghosh and Gagnon (1989) | | | | | | | | | Literature review | | | | | | |
| Magnanti and Vachani (1990) | | ✓ | ✓ | | | ✓ | ✓ | | ✓ | | | | ✓ | | B&C |
| Potts and Van Wassenhove (1992) | | | | | | | | | Literature review | | | | | | |
| Uzsoy et al. (1992) | | | | | | | | | Literature review | | | | | | |
| Kim and Kim (1996) | | ✓ | | ✓ | | ✓ | ✓ | | ✓ | | | | ✓ | | SA + GA |
| Franck et al. (1997) | ✓ | | ✓ | | | ✓ | ✓ | | ✓ | | | | | ✓ | – |
| Lam and Xing (1997) | | | | | | | | | Literature review | | | | | | |
| Lee et al. (1997) | | | | | | | | | Literature review | | | | | | |
| Ierapetritou and Floudas (1998) | | ✓ | | ✓ | | ✓ | ✓ | | | ✓ | | | | ✓ | GAMS |
| Khouja et al. (1998) | | ✓ | ✓ | | | ✓ | | ✓ | | ✓ | | | ✓ | | GA |
| Ozdamar and Birbil (1998) | | ✓ | | ✓ | | ✓ | ✓ | | ✓ | | | | ✓ | | SA + TS + GA |
| Alidaee and Womer (1999) | | | | | | | | | Literature review | | | | | | |
| Cheng et al. (1999) | | | | | | | | | Literature review | | | | | | |
| Ierapetritou et al. (1999) | | ✓ | ✓ | | | ✓ | ✓ | | | ✓ | | | | ✓ | GAMS |
| Jain and Meeran (1999) | | | | | | | | | Literature review | | | | | | |
| Bruggemann and Jahnke (2000) | | ✓ | ✓ | | | ✓ | ✓ | | ✓ | | | | ✓ | | SA |
| El-Hafsi (2000) | | ✓ | ✓ | | | ✓ | | ✓ | | ✓ | | | ✓ | | Heuristic |
| Meyr (2000) | | ✓ | ✓ | | ✓ | | ✓ | | ✓ | | | | ✓ | | TA + SA |
| Schwindt and Trautmann (2000) | | ✓ | | ✓ | ✓ | | ✓ | | | ✓ | ✓ | | | | Heuristic |
| Ouenniche and Bertrand (2001) | | ✓ | ✓ | | | ✓ | ✓ | | | ✓ | | | ✓ | | Heuristic |
| Ouenniche and Boctor (2001) | | ✓ | ✓ | | | ✓ | | ✓ | | ✓ | | | ✓ | | Heuristic |
| Sung and Min (2001) | | ✓ | ✓ | | | ✓ | ✓ | | | ✓ | | ✓ | | | Heuristic |
| Harjunkoski and Grossmann (2002) | | ✓ | ✓ | | | ✓ | ✓ | | | ✓ | | | ✓ | | ILOG |
| Nozick et al. (2002) | ✓ | | ✓ | | | ✓ | ✓ | | | ✓ | | | ✓ | | Heuristic |
| Karimi et al. (2003) | | | | | | | | | Literature review | | | | | | |
| Floudas and Lin (2004) | | | | | | | | | Literature review | | | | | | |
| Rocha et al. (2004) | | ✓ | ✓ | | ✓ | | ✓ | | | ✓ | | | | ✓ | B&B + GRASP |
| Alvarez-Valdes et al. (2005) | | ✓ | | ✓ | | ✓ | ✓ | | ✓ | | | | ✓ | | Heuristic |
| Stadtler (2005) | | ✓ | | ✓ | | ✓ | ✓ | | ✓ | | | | ✓ | | – |
| Stevenson et al. (2005) | | | | | | | | | Literature review | | | | | | |
| Jodlbauer (2006) | | ✓ | ✓ | | | ✓ | ✓ | | | ✓ | | | ✓ | | Theorems |
| Tavakkoli-Moghaddam et al. (2006) | | ✓ | ✓ | | | ✓ | ✓ | | | ✓ | | ✓ | | | B&B |
| Torabi et al. (2006) | | ✓ | ✓ | | | ✓ | ✓ | | | ✓ | | | ✓ | | GA |
| Zhu and Wilhelm (2006) | | | | | | | | | Literature review | | | | | | |
| Chen and Ji (2007) | | ✓ | | ✓ | | ✓ | | ✓ | | ✓ | | | ✓ | | CPLEX |
| Jans and Degraeve (2007) | | | | | | | | | Literature review | | | | | | |
| Shabtay and Steiner (2007) | | | | | | | | | Literature review | | | | | | |
| Tang and Liu (2007) | ✓ | | ✓ | | | ✓ | ✓ | | ✓ | | | | | ✓ | Decomposition + LR |
| Tasgetiren et al. (2007) | ✓ | | ✓ | | | ✓ | | ✓ | | ✓ | ✓ | | | ✓ | PSO + VNS |
| Xuan and Tang (2007) | | ✓ | ✓ | | | ✓ | ✓ | | ✓ | | | | ✓ | | DP |
| Chan et al. (2008) | | ✓ | | ✓ | ✓ | | ✓ | | | ✓ | | ✓ | | | GA |
| Defersha and Chen (2008) | | ✓ | | ✓ | | ✓ | ✓ | | ✓ | | | | ✓ | | GA + LP |
| Huang and Yao (2008) | | ✓ | ✓ | | | ✓ | | ✓ | | ✓ | | | ✓ | | GA |
| Monkman et al. (2008) | | ✓ | | ✓ | ✓ | | ✓ | | | ✓ | | | ✓ | | GRASP |
| Raza and Akgunduz (2008) | | ✓ | ✓ | | | ✓ | | ✓ | | ✓ | | | ✓ | | SA |
| Salvietti and Smith (2008) | | ✓ | ✓ | | | ✓ | | ✓ | | ✓ | | | ✓ | | Heuristic |
| Brander and Segerstedt (2009) | | ✓ | ✓ | | | ✓ | | ✓ | | ✓ | | | ✓ | | Heuristic |
| Buscher and Shen (2009) | | ✓ | ✓ | | | ✓ | | ✓ | | ✓ | ✓ | | | | TS |
| Luo et al. (2009) | | ✓ | ✓ | | ✓ | | ✓ | | ✓ | | ✓ | | | | GA |
| Maravelias and Sung (2009) | | | | | | | | | Literature review | | | | | | |
| Pan and Yang (2009) | | ✓ | ✓ | | ✓ | | ✓ | | | ✓ | | ✓ | ✓ | | Heuristic |

(*continued on next page*)