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Decision Support

Competition among non-life insurers under solvency constraints: A game-theoretic approach

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ABSTRACT

We formulate a noncooperative game to model competition for policyholders among non-life insurance companies, taking into account market premium, solvency level, market share and underwriting results. We study Nash equilibria and Stackelberg equilibria for the premium levels, and give numerical illustrations.

1. Introduction

Insurance pricing is a classical topic for both actuaries and academics. Standard actuarial approaches for non-life insurance typically suggest to use expectation, standard deviation, or quantiles of the underlying risk to determine a suitable premium. For an overview of principles of premium calculation, see e.g. Teugels and Sundt (2004). The resulting (so-called technical) premium is then often altered by marketing and management departments, and actual deviations from the technical premium can be considerable. Affordability by customers and mutualization across the portfolio are often used as arguments to justify that policyholders do not necessarily pay the risk-based premium. But another important reason for such deviations from the technical premium is the dependency on market conditions. In order to study that factor, a market model is needed to study the economic interactions between insurers and policyholders.

Basic economic models suggest that the equilibrium premium is the marginal cost, as any upward deviation from this premium equilibrium will result in losing all the policies in the next period. Other advanced economic models generally focus on moral hazard and adverse selection. The Rothschild and Stiglitz's model (see Rothschild and Stiglitz (1976)) deals with a utility-based agent framework where policyholders have private information on their own risk. In this model, insurers provide a menu of contracts, i.e. pairs of premium and deductible, from which policyholder can freely choose. At the equilibrium, individuals with low risk aversion choose full coverage, whereas individuals with high risk aversion are more attracted to partial coverage. Note that an equilibrium price may not exist if all insurers offer just one type of contract. Picard (2009) considers an extension by allowing insurers to offer participating contracts (such as mutual-type contracts). This feature guarantees the existence of an equilibrium, which reveals the risk level of the policyholders. An important area of applications for such models is health insurance, where moral hazard and adverse selection play a major role, see e.g. Geoffard et al. (1998), Wambach (2000) and Mimra and Wambach (2010). But, in practice customers do not move from one insurer to a cheaper one as swiftly as economic models anticipate. The inertia of the insurance demand prevents policyholders to always look for the cheapest insurer, see Smith et al. (2000) for a case study in Australia. Accordingly, the customer behavior is much more complicated.

Moreover, the economic models mentioned above are not able to incorporate some insurance market features. Taylor (1986, 1987) deals with underwriting strategies of insurers and provides first attempts to model optimal responses of an insurer to the market on a given time horizon, see also Kliger and Levikson (1998); Emms et al. (2007); Moreno-Codina and Gomez-Alvado (2008) for extensions. All these papers focus on one single insurer and

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in that way assume that insurers are playing a game against an impersonal market player, so that the market price is independent of their own actions.

In this paper, we want to further investigate the suitability of game theory for insurance market modeling. The use of game theory in actuarial science has a long history dating back to K. Borch and J. Lemaire, who mainly used cooperative games to model risk transfer between insurer and reinsurer, see e.g. Section 3.1 of Brockett and Xia (1995) for a review. Among earlier work using noncooperative game theory to model the non-life insurance market, two kinds of models were pursued: the Bertrand oligopoly where insurers set premiums and the Cournot oligopoly where insurers choose optimal values of insurance coverage. Polborn (1998) considers a Bertrand model in which rational consumers maximize their utility function and for which the equilibrium premium is the expected loss. Rees et al. (1999) extend this model by requiring insurers to announce both a premium and a capital value. Under rational behavior, the premium equilibrium remains the expected loss and the capital equilibrium avoids insolvency. Powers and Shubik (1998) propose a Cournot model with two types of players. Policyholders state the amount that they are willing to pay, and insurers state the amount of risk they are willing to underwrite. Based on a clearing-house system to determine the market price, each player maximizes its expected utility. Assuming risk neutral insurers and risk averse consumers, the resulting premium equilibrium is larger than the expected loss. They also study scale effects of the number of insurers on the premium equilibrium. Powers and Shubik (2006) include reinsurers as additional players and study the optimal number of reinsurers in an insurance market.

The present paper aims to model competition in non-life insurance markets with noncooperative game theory in order to extend the insurer-vs-market reasoning of Taylor (1986, 1987). We extend the Bertrand model of Rees et al. (1999) by considering a lapse model and an aggregate loss model for policyholders. The lapse model describes the policyholder behavior through a lapse probability which is a function of the premiums offered by the insurers. We also consider a solvency constraint function for insurers. As a main result, we show that incorporating competition when setting premiums leads to a significant deviation of Nash and Stackelberg equilibria both from the actuarial premium and a one-insurer optimized premium.

The rest of the paper is organized as follows. Section 2 develops the one-period noncooperative game of this paper. Existence and uniqueness of a premium equilibrium are established. Section 3 presents numerical illustrations of the game. A conclusion and perspectives are given in Section 4.

2. A one-period model

Consider I insurers competing in a market of n policyholders with one-year contracts (n is fixed). The policyholders are assumed to react to price changes (either stay with the present insurer or switch to one of the competitors), but do not have any other influence on the premium level (which is a realistic assumption, in particular for personal lines of business such as compulsory third-party motor liability). In view of the one-year time horizon and the randomness of claim sizes, this model focuses on non-life insurance products (i.e. products for which the claim event is *not* linked to the life of the policyholder).

The "game" for insurers is to set the premium for which policies are offered to the policyholders. Let $(x_1, \ldots, x_I) \in \mathbb{R}^I$ be a price vector, with x_j representing the premium of Insurer j. Once the premium is set by all insurers, the policyholders choose to renew or to lapse from their current insurer. Then, insurers pay occurring claims during the coverage year. At the end of the period, underwriting results are determined, and the insurer capital is updated: some insurers may be bankrupt. As we deal with a one-period model, for simplicity we do not consider investment results.

In the next subsections, we present the four components of the game: (i) a lapse model, (ii) a loss model, (iii) an objective function and (iv) a solvency constraint function. These four components are critical factors for the analysis of the non-life insurance market, see e.g. IASB (2008). In the sequel, a subscript $j \in \{1, \ldots, I\}$ will always denote an insurer index, whereas a subscript $i \in \{1, \ldots, n\}$ denotes policyholder index. In the sequel, "insurer" is used when referring to players of the insurance game.

2.1. Lapse model

In this subsection, we present our lapse model which is designed as a compromise between reflecting the policyholders' behavior in a reasonable way, yet keeping mathematical tractability. Let n_j be the initial portfolio size of Insurer j (such that $\sum_{j=1}^{I} n_j = n$). It seems natural that the choice of policyholders for an insurer is highly influenced by the choice of the previous period. We assume that the dispatch (among the I insurers) of the n_j policyholders of Insurer j follows an I-dimensional multinomial distribution $\mathcal{M}_I(n_j, p_{j\rightarrow}(x))$ with probability vector $p_{j\rightarrow}(x) = (p_{j\rightarrow 1}(x), \ldots, p_{j\rightarrow I}(x))$. The probability $p_{j\rightarrow k}(x)$ to move from Insurer j to Insurer k naturally depends on the price vector x, (concretely, the difference of premiums). Empirically, the probability to lapse $p_{j\rightarrow k}(x)$ (with $k \neq j$) is generally much lower than the probability to renew $p_{j\rightarrow j}(x)$. To our knowledge, only the UK market shows lapse rates above 50%, cf. Dreyer (2000).

In the economics literature, $p_{j\rightarrow k}$ is considered in the framework of discrete choice models. In the random utility maximization setting, McFadden (1981, chap. 5) or Anderson et al. (1989) propose multinomial logit and probit probability choice models. In this paper, we choose a multinomial logit model, because of its simplicity (the probit link function, based on the multivariate normal distribution, would not significantly change the shape of the lapse function). Working with unordered choices, we arbitrarily set the insurer reference category for $p_{j\rightarrow k}$ to j, the current insurer. We define the probability for a customer to go from insurer j to k given the price vector x by the multinomial logit model

$$p_{j\rightarrow k}(x) = \begin{cases} \frac{1}{1+\sum_{l\neq j} e^{f_j(x_j,x_l)}} & \text{if } j = k, \\ \frac{e^{f_j(x_j,x_k)}}{1+\sum_{l\neq j} e^{f_j(x_j,x_l)}} & \text{if } j \neq k, \end{cases} \tag{1}$$

where the sum is taken over the set of insurers $\{1, \ldots, I\}$ and f_j is a price-sensitivity function. We consider two types of price functions

$$\bar{f}_j(x_j, x_l) = \bar{\mu}_j + \bar{\alpha}_j \frac{x_j}{x_l} \text{ and } \tilde{f}_j(x_j, x_l) = \tilde{\mu}_j + \tilde{\alpha}_j (x_j - x_l). \tag{2}$$

The first function $\bar{f}_j$ assumes a price-sensitivity according to the ratio of proposed premium x_j and competitor premium x_l, whereas $\tilde{f}_j$ works with the premium difference $x_j - x_l$. Parameters μ_j,α_j represent a base lapse level and price-sensitivity, respectively. We assume that insurance products display positive price elasiticity of demand $\alpha_j > 0$. One can check that $\sum_k p_{j\rightarrow k}(x) = 1$.

Eq. (1) can be rewritten as

$$p_{j\rightarrow k}(x) = p_{j\rightarrow j}(x)(\delta_{jk} + (1 - \delta_{jk})e^{f_j(x_j,x_k)}),$$

with δ_{ij} denoting the Kronecker delta. It is difficult to derive general properties of the distribution of a sum of binomial variables with different probability parameters, except when the size parameters n_j are reasonably large, in which case the normal approximation is appropriate.