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Discrete Optimization

Construction heuristics for two-dimensional irregular shape bin packing with guillotine constraints

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ABSTRACT

The paper examines a new problem in the irregular packing literature that has many applications in industry: two-dimensional irregular (convex) bin packing with guillotine constraints. Due to the cutting process of certain materials, cuts are restricted to extend from one edge of the stock-sheet to another, called guillotine cutting. This constraint is common place in glass cutting and is an important constraint in two-dimensional cutting and packing problems. In the literature, various exact and approximate algorithms exist for finding the two dimensional cutting patterns that satisfy the guillotine cutting constraint. However, to the best of our knowledge, all of the algorithms are designed for solving rectangular cutting where cuts are orthogonal with the edges of the stock-sheet. In order to satisfy the guillotine cutting constraint using these approaches, when the pieces are non-rectangular, practitioners implement a two stage approach. First, pieces are enclosed within rectangle shapes and then the rectangles are packed. Clearly, imposing this condition is likely to lead to additional waste. This paper aims to generate guillotine-cutting layouts of irregular shapes using a number of strategies. The investigation compares three two-stage approaches: one approximates pieces by rectangles, the other two approximate pairs of pieces by rectangles using a cluster heuristic or phi-functions for optimal clustering. All three approaches use a competitive algorithm for rectangle bin packing with guillotine constraints. Further, we design and implement a one-stage approach using an adaptive forest search algorithm. Experimental results show the one-stage strategy produces good solutions in less time over the two-stage approach.

1. Introduction

There exist in the literature a high volume and variety of investigations into two-dimensional (2D) cutting and packing problems, which reflects the large application scope, such as ship building, shoe manufacturing, garment manufacturing and tool manufacturing, and a range of materials, for example glass, metal, wood, and textiles. Within these publications, a popular focus of research is generating cutting patterns that satisfying guillotine cutting constraints. These constrain any cut to begin at one edge of the stock sheet and continue in a straight line to another edge of the stock sheet. To the best of our knowledge, all of the algorithms are designed for solving 2D rectangular shape cutting problems where all cuts are orthogonal to the edges of the stock-sheet. Guillotine cutting with irregular pieces has not been tackled directly. In this problem, guillotine cuts are not constrained to be orthogonal to the rectangular stock sheet edges, and pieces can be continuously rotated.

An example of the irregular shape bin packing problem with guillotine constraints arises from the glass cutting industry and in particular the manufacture of conservatories (glass houses). Although many of the pieces are rectangular, there is a substantial number of irregular pieces. These are convex polygons with up to five sides in general, and occasionally more. It is common for conservatories to be a bespoke design (usually based on a standard style) to fit the specific building, hence, glass is cut to order. To satisfy the guillotine cutting constraints in practice, items of irregular shapes are individually, or in pairs, enclosed within rectangles and these rectangles are then arranged into a cutting pattern. This adds a restriction that is not present in practice, which is likely to create patterns with more waste than necessary.

In this paper, we implement four pattern generation strategies with the objective of minimizing the number of bins required to pack all items. Primarily, we aim to investigate the benefit of generating cutting patterns which satisfy the guillotine cutting constraint by implementing the cuts directly on the irregular shapes instead of on rectangle enclosures. This one-stage approach is based on an efficient forest search algorithm. The forest constructs multiple layouts in parallel according to a dynamic measure of the quality of the

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partial layout. In order to benchmark our approach we implement a state of the art rectangle guillotine cutting algorithm of Charalambous and Fleszar [6] and generate solutions by approximating each piece by its minimum area enclosing rectangle. Further, we attempt to improve on the two stage approach used in practice by using phi-functions to cluster all pairs of pieces in their minimum rectangle enclosure and use a greedy heuristic to select a subset to pack, again using the approach of Charalambous and Fleszar [6].

The contributions of this paper are many. We have brought a new problem to the literature that is found in practice. As a result, there is significant scope for further research. We have designed an efficient search heuristic using a dynamic solution evaluation function. The approach can handle continuous rotation of the pieces, multiple bins, and guillotine cuts. Irregular shape packing usually constrains the number of orientations of the pieces, approaches generally only pack a single strip and to our knowledge formulations have never included guillotine constraints. Further, the paper describes a second approach, based on industry practice, but using state of the art techniques to solve the problem by first optimizing the rectangle enclosure of pairs of pieces using phi-functions and then packing using a rectangle guillotine packing approach. This in itself is new to the literature. Finally, we have also introduced new benchmark data sets for this problem.

In the next section (Section 2), we give a more detailed description of the problem. In Section 3 we review some related literature on guillotine bin packing. Section 3 explains the one-stage approach, including some notation and definitions to describe the important characteristics of the problem, and details of a core function, best match, that forms the basis of the algorithm. It also includes a description of the forest search algorithm. In Section 5, we describe the two-stage approach based on the work of Charalambous and Fleszar [6], minimum rectangle clustering based on phi-functions and our greedy selection. Section 6 contains the computational study and the discussion of the results. Finally, in Section 7 we present the conclusions.

2. Problem description

The problem objective is to cut all demand pieces from the minimum number of stock sheets possible, hence it is an input minimization problem. There are sufficient standard size rectangular stock-sheets available to meet demand, where the stock sheet has length L and width W. The demand set D contains N irregular shaped pieces, where each piece is considered to be unique and the demand of each piece is one. According to the typology proposed by Wäscher et al. [16] this is a single bin size bin packing problem (SBSBPP).

Further refinements to the problem type are that all pieces are convex, and usually irregular. Pieces can be rotated continuously i.e. there are no fixed rotation angles. Further, the stock sheet can be rotated. In principle this is taken care of by rotating the pieces. However, in Charalambous and Fleszar [6] the orientation of a non-square stock sheet is important. Only guillotine cuts are allowed. A guillotine cut is a single straight line cut that begins at an edge of the stock-sheet and ends at another edge. Unlike the vast majority of the literature, the cutting line is not constrained to be parallel to an edge of the stock-sheet. Often when considering guillotine constraints, pieces must be cut free from the stock sheet with a maximum number of cuts, which is typically three. In this problem there are no limits on the number cuts.

3. Literature review

There are three key components of the problem under consideration: bin packing, guillotine cuts, and irregular shapes. To our knowledge there are no papers that tackle these three together. In addition, irregular shape packing literature is almost exclusively strip packing (single infinite length stock sheet) with a finite fixed set of rotation angles. Those who have tackled multiple stock sheet problems reduce the problem to a one-dimensional cutting stock problem using pre-defined pattern layouts, for example, Degraeve and Vandebroek [8]. A key challenge in packing irregular shapes is handling complex geometry, particularly when pieces contain concavities. In this paper all pieces are convex. Instead, the key challenge arises in modelling efficiently continuous rotation of the pieces, which is not commonly dealt with in the literature. For a discussion of techniques for handling the geometry in irregular shape packing see Bennell and Oliveira [4]. Aside from the geometry, solution approaches to irregular packing are almost all heuristic and can be divided into those that build up to a final solution through sequentially adding to partial solutions, and those that work with complete solutions and search by making small changes to the incumbent solution. The latter approach can be subdivided into representing the solution by a sequence, or packing order, that is decoded by a construction heuristic, or by the co-ordinate positions of the pieces in the layout. For a review of solution approaches to the irregular packing problem see Bennell and Oliveira [3].

The rectangle bin packing problem with guillotine constraints has the most similarities to the problem we are tacking in this paper. Lodi et al. [12] survey two dimensional rectangle bin packing, including algorithms that handle guillotine constraints. They describe the one- and two-phase approach, where both consist of packing pieces onto shelves along the width of the bin. The former directly packs pieces into the bin, where as the latter optimizes the packing of the shelves into the bins by modelling as a one dimensional bin packing problem. Lodi et al. [10] creates the shelves by solving a series of 0–1 knapsack problems improving on the performance of the finite first fit and finite best strip heuristics of Berkey and Wang [5]. More recent construction heuristics are not constrained to creating shelves. Charalambous and Fleszar [6] start by generating simple patterns, initially across the width of the bin, and subsequently within free rectangle areas. Pieces may shift horizontally or vertically in order to maximize the size of the free rectangle, while maintaining the guillotine constraint. Fleszar [9] propose a constructive heuristic where the insertion decision is made by first-fit, best-fit or critical-fit criteria. Patterns are generated using a tree structure where nodes determine the cuts and the leaf nodes are pieces. Further improvement is made by a justification heuristic. Polyakovsky and M'Hallah [13] modify the well know bottom left construction heuristic to meet guillotine constraints. After placing each piece, they apply both horizontal and vertical guillotine cuts and select the one that gives the largest rectangle area available for packing. Pieces are assigned to bins using an agent-based algorithm. Pieces may be agent-initiators attracting individual-agents (pieces) to their group in order to maximize the fitness of the group. Individual-agents compete to join groups to maximize their purpose parameters. These groups are assigned to the same bin and arranged using the guillotine bottom left heuristic. Lodi et al. [11] uses tabu search to assign pieces to bins. Initially, one piece is packed in each bin. The heuristic attempts to empty weak bins by assigning pieces to sub-instances that include the pieces from k bins. Instead of assigning pieces to bins and then packing, Alvelos et al. [1] define a sequence for packing the pieces while keeping a list of candidate locations for the next piece. The solution is improved using variable neighborhood descent where moves are made within the packing sequence. Although none of these papers directly apply to the problem addressed in this paper, we have made use of some core knowledge. We adopt methodologies for efficiently processing the geometry of irregular convex shapes with free rotation using some classic concepts and phi-functions. We follow the common theme of