European Journal of Operational Research 230 (2013) 539–550

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Two-stage vehicle routing problem with arc time windows: A mixed integer programming formulation and a heuristic approach

Cihan Çetinkaya [a], Ismail Karaoglan [b,*], Hadi Gökçen [c]

[a] Logistics Command of Gendarmerie, Guvercinlik, Ankara, Turkey
[b] Department of Industrial Engineering, Faculty of Engineering and Architecture, Selcuk University, 42075 Selcuklu, Konya, Turkey
[c] Department of Industrial Engineering, Faculty of Engineering, Gazi University, 06570 Maltepe, Ankara, Turkey

ARTICLE INFO

Article history:
Received 26 August 2011
Accepted 2 May 2013
Available online 11 May 2013

Keywords:
Logistics
Vehicle routing problem with arc time windows
Memetic algorithm
Mathematical formulation

ABSTRACT

In this paper, we introduce a new variant of the Vehicle Routing Problem (VRP), namely the Two-Stage Vehicle Routing Problem with Arc Time Windows (TS_VRP_ATWs) which generally emerges from both military and civilian transportation. The TS_VRP_ATW is defined as finding the vehicle routes in such a way that each arc of the routes is available only during a predefined time interval with the objective of overall cost minimization. We propose a Mixed Integer Programming (MIP) formulation and a heuristic approach based on Memetic Algorithm (MA) to solve the TS_VRP_ATW. The qualities of both solution approaches are measured by using the test problems in the literature. Experimental results show that the proposed MIP formulation provides the optimal solutions for the test problems with 25 and 50 nodes, and some test problems with 100 nodes. Results also show that the proposed MA is promising quality solutions in a short computation time.

1. Introduction

The design of distribution systems is one of the key components of logistics systems as it offers a great potential in order to reduce costs and to improve service quality. Therefore, huge number of studies have been devoted to find good solutions for these systems. In this research area, one of the main topics is the vehicle routing problem (VRP) which involves the design of a set of minimum cost routes serving a set of customers with known demands. In classical VRPs, each customer is served exactly once, each vehicle originates and terminates at the main depot, and all customers must be assigned to only one vehicle such that the vehicle capacities are not exceeded. Following the introduction of the VRP by Dantzig and Ramser (1959), several mathematical models and exact solution procedures developed for small and medium-size VRPs in the literature (Achuthan et al., 2003; Araque et al., 1994; Augerat et al., 1998; Baldacci et al., 2012; Lysgaard et al., 2004; Ralphs, 2003). It is commonly known that VRP belongs to the class of NP-hard problems. Because of its intractability manner, a great deal of heuristic/meta-heuristic approaches have been successfully implemented for the problem. We refer the readers to the review of (Toth and Vigo, 2002) for VRPs and their applications, (Eksioglu et al., 2009) for taxonomic review of VRPs, (Funke et al., 2005) for local search algorithms, and more recently (Doerner and Schmid, 2010) for meta-heuristic algorithms.

The vehicle routing problem with time windows (VRP_TW) is a natural extension of the VRP, motivated by the availability of nodes for a specific time windows. In general form of the VRP_TW, each node must be visited in a predetermined time interval. However, in practice, time window restrictions for node routing problems (i.e. VRPs) can also be appeared on arcs in addition to nodes. Two of them can be given as follows:

- In military logistic system of Turkey, some roads can only be used in predetermined time windows. For instance, especially in eastern cities of Turkey, the roads between city centers and districts generally pass through highlands which are potential ambush locations. Therefore, these roads can only be used in daylights to avoid terrorist attack.
- In addition to the military applications, time window restriction can also be appeared in civilian transportation systems. For example, the trucks cannot enter the city center in rush hours (i.e. between 7–10 am and between 4–7 pm), because these hours includes the beginning and ending time of shifts and the traffic density reaches at the extreme level. Therefore, the entrance of trucks to the city center in these hours is forbidden by local government to facilitate the traffic flow.

Although, there exist many studies in the node routing literature considering the node time windows, to the best of

* Corresponding author. Tel.: +90 332 223 3751.
E-mail addresses: cihancetinkaya1903@hotmail.com (C. Çetinkaya), ikaraoglan@selcuk.edu.tr (I. Karaoglan), hgokcen@gazi.edu.tr (H. Gökçen).

0377-2217/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2013.05.001

our knowledge, there is no previous study considering the arc time windows. Because of the increasing importance of these restrictions for the military and civilian transportation, this paper considers the Vehicle Routing Problem with Arc Time Windows (VRP_ATWs). Two-stage version of VRP_ATW (namely TS_VRP_ATW) is discussed as well, which is the distribution network structure of the military logistic system in Turkey. In this problem, the network is divided into three layers (facility, depots and customers) and routing operations encountered in both stages each consisting successive layers (i.e. between facility-depots, and depots-customers). We propose Mixed Integer Programming (MIP) formulations for both VRP_ATW and TS_VRP_ATW. We also develop a meta-heuristic approach called Memetic Algorithm (MA) to solve the large-size problem instances. In order to evaluate the performance of the proposed MIP formulation and MA, we carry out experimental study by using various size test problems adapted from the literature. Experimental results show that the proposed MIP formulation gives the optimal solutions for the test problems with 25 and 50 nodes, and some test problems with 100 nodes. The proposed MA also achieves good quality solutions especially for the large-size test problems. The average percentage gap values of the proposed MIP formulation and MA are 1.05 and 2.26, while the average solution times are 984.81 and 11.20 s, respectively.

The main contribution of this paper is threefold. First, we define the VRP_ATW and emphasize the relationship with other node routing problems. Second, we propose a MIP formulation for VRP_ATW, and extend this formulation to TS_VRP_ATW. Although, there are several papers considering complex routing operations among stages (i.e. (Ambrosino and Graziascutella, 2005; Ambrosino et al., 2009)), to the best of our knowledge, this is the first paper dealing with multi stage VRP under time window restrictions. Finally, we propose a meta-heuristic approach (MA) to solve the large-size problem instances.

The rest of this paper is organized as follows. Related literature for the problem given in Section 2. The definition of the VRP_ATW, the mathematical formulation and the relationship with other routing problems are shown in Section 3. The definition and the mathematical formulation for the TS_VRP_ATW are given in Section 4. The detailed description of the proposed algorithm is given in Section 5. Section 6 reports computational results, and conclusion and future directions are discussed in the last section.

2. Literature review

Hadjar and Soumis (2009) have studied the closely related problem, namely multiple depot vehicle scheduling problem with time windows, and proposed a branch-and-price algorithm for the problem. In this paper, the time windows have been transferred from nodes to the arcs for reducing the dimension of huge number of variables, and thus speed up the solution procedure. However, such a problem involves node time windows rather than arc time windows under different structure.

The concept of "time window restrictions on arcs" has been well documented in the capacitated arc routing problem (CARP) literature. In these problems, the main objective is to find a least cost traversal of all required edges in the network in such a way that service must begin and end at the same node (i.e. depot node), and the service must start and finish in a predetermined time interval for all required edges. These time window restrictions may be encountered in areas such as urban waste collection, postal deliveries, treatment of roads to remove snow or to prevent ice forming, or taking pictures of street and roads for digital maps. CARP with time windows (CARP_TW) have been introduced by Mullaseril (1997). In this paper, a heuristic algorithm and a transformation approach into a VRP_TW have been developed. Wøhlk (2005) has also considered the CARP_TW and presented two mathematical models, several heuristics, a column generation technique to obtain the lower bounds and a dynamic programming approach for the problem. Reghioui et al. (2007) have proposed a hybrid heuristic technique combining greedy randomized adaptive search procedure (GRASP) and path relinking. The authors have shown that proposed heuristic can compete with the best existing heuristic in the literature. Vansteenwegen et al. (2010) have considered the mobile mapping van problem equivalent to CARP with time soft windows and proposed a two-phase heuristic search algorithm for the problem. The authors have proven the strength of the algorithm on the test problems derived from the literature, and several real-life examples. For reference, Corbeŕn and Prins (2010) can be reviewed.

3. Vehicle routing problem with arc time windows

In this section, we first define the VRP_ATW and give the relationship with other related problems. We then present a Mixed Integer Programming (MIP) formulation for the problem.

3.1. Problem definition

The VRP_ATW can be defined as follows: Let $G = (N,A)$ be a complete directed graph where $N = \{0\} \cup N_C$ is a set of nodes in which "0" and N_C represent the depot and customers, respectively, and $A = \{(i,j) | i \neq j,\ \forall i,\ j \in N\}$ is the set of arcs. A fleet of homogeneous vehicles with known capacity is available at the depot. Each customer has a nonnegative demand. Each arc has a nonnegative cost, a transportation time, and a time window which identifies the availability of arc. The transition for all arcs must begin within its time window, if used. The problem is to determine the vehicle routes with minimum total cost under following constraints: (i) each vehicle is used at most one route, (ii) each customer is served exactly once, (iii) each route begins and ends at the depot, (iv) the total vehicle load at any point of the route does not exceed the vehicle capacity, and (v) the transition for each arc on any route begins within its time window.

3.2. Mixed integer programming formulation for the VRP_ATW

We next introduce sets, notations and decision variables then proceed by representing the MIP formulation (referred as F1).

3.2.1. Sets and notations

N_C: set of all customers
N: set of all nodes ($N = \{0\} \cup N_C$)
Q: vehicle capacity
d_i: demand of customer $i \in N_C$ with $d_i \leqslant Q$
c_{ij}: cost of an arc $(i,j) \in A$
t_{ij}: transportation time of an arc $(i,j) \in A$
a_{ij}: earliest available time of an arc $(i,j) \in A$
b_{ij}: latest available time of an arc $(i,j) \in A$

3.2.2. Binary decision variables

$$x_{ij} = \begin{cases} 1 & \text{if a vehicle travels directly from node } i \text{ to node } j \quad (\forall i,j \in N) \\ 0 & \text{otherwise} \end{cases}$$