ARTICLE IN PRESS

European Journal of Operational Research xxx (2013) xxx–xxx

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Incorporating performance measures with target levels in data envelopment analysis

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ARTICLE INFO

Article history:
Received 12 December 2012
Accepted 23 April 2013
Available online xxxx

Keywords:
Data envelopment analysis (DEA)
Performance measure
Target level
Efficiency

ABSTRACT

Data envelopment analysis (DEA) is a technique for evaluating relative efficiencies of peer decision making units (DMUs) which have multiple performance measures. These performance measures have to be classified as either inputs or outputs in DEA. DEA assumes that higher output levels and/or lower input levels indicate better performance. This study is motivated by the fact that there are performance measures (or factors) that cannot be classified as an input or output, because they have target levels with which all DMUs strive to achieve in order to attain the best practice, and any deviations from the target levels are not desirable and may indicate inefficiency. We show how such performance measures with target levels can be incorporated in DEA. We formulate a new production possibility set by extending the standard DEA production possibility set under variable returns-to-scale assumption based on a set of axiomatic properties postulated to suit the case of targeted factors. We develop three efficiency measures by extending the standard radial, slacks-based, and Nerlove–Luenberger measures. We illustrate the proposed model and efficiency measures by applying them to the efficiency evaluation of 36 US universities.

1. Introduction

Data envelopment analysis (DEA) is an approach for evaluating the relative efficiency of a set of (homogeneous) peer entities, called Decision-Making Units (DMUs), whose performance is characterized by a set of multiple performance measures. In DEA, these multiple performance measures are classified into inputs and outputs. Based upon the observed values on the multiple inputs and multiple outputs, DEA determines or estimates a production frontier (or best-practice frontier) of the underlying technology. For a more detailed discussion on DEA models and applications, the reader is referred to Cooper et al. (2011).

In conventional DEA models, it is generally assumed that a higher output level and a lower input level are preferred. Therefore, one DMU is deemed more efficient than another if that DMU produces larger amounts of outputs using the same amounts of inputs, or produces the same amounts of outputs using smaller amounts of inputs. Generally, inputs are of *smaller-the-better* type and outputs are of *larger-the-better* type in conventional DEA models. We refer to these types of inputs and outputs as *regular*, as they do not need any transformation or special treatment prior to DEA application. DEA inputs and outputs that require special treatment include undesirable factors. For example, pollution is an undesirable output and needs to be reduced (Seiford and Zhu, 2002). However, despite of the special treatment, these performance measures can still be classified as outputs or inputs.

Our current study is motivated by situations where performance measures can be classified as neither inputs nor outputs. Such performance measures have target levels, and every DMU strives to achieve the target levels for that type of factors. If a performance measure is below its target level for some DMUs, that measure needs to be increased and should be treated as an output. However, if the measure is above the target level for other DMUs, that measure needs to be decreased and should be treated as an input. The standard assumption made in conventional DEA models for inputs and outputs does not hold for these types of performance measures. We will refer to this type of factors as *targeted*. For example, suppose we are comparing and evaluating different kinds of diets or animal feeds. In addition to the cost of each diet or animal feed (which can be treated as inputs), we have measures such as the amount of protein, the mount of vitamin C, and the ratio of fat to carbohydrate, among others, contained in each diet or animal feed. The amounts of protein and vitamin C usually have generally accepted target levels, e.g., 5 grams of protein and 100 milligrams of vitamin C, no more and no less. Furthermore, it is typically required that the ratio of fat to carbohydrate should

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http://dx.doi.org/10.1016/j.ejor.2013.04.051

be close to certain desirable target level, e.g., the ratio of total fat to carbohydrate contained in a diet should be around 1:3. As a result, these measures cannot be classified as either inputs or outputs in a single DEA run. Another example can be found in product and process evaluation. Most parts in mechanical fittings have designed (or desirable) dimensions and characteristics such as thickness, length, and density. Furthermore, products and processes typically have certain performance standards or benchmarks, and any variations to the standards are undesirable. Therefore, when several product and process designs are competing with each other to be chosen and implemented, a DEA model which can handle targeted measures or factors is required. Other examples of targeted factors include journal acceptance rate which is a factor affecting the journal backlog and quality reputation. Performance evaluation with consistency-based performance measure (*i.e.*, for measuring consistent quality) is another example. For example, there is a target value for the specification of "door seal resistance" in the House of Quality proposed by Hauser and Clausing (1988). In ice cream production and packaging, there are both cost and government regulations that require the ice cream weight meets the specifications, and any deviation (either below or above) can result in customer complaints/lost sale or increased cost.

Such factors with targeted levels are called "targeted response" in Liu et al. (2006). Although Liu et al. (2006) point to the fact that such type of factors should be treated properly, they do not provide a model for dealing with it. Cook et al. (2006) and Cook and Zhu (2007) discuss performance measures called "flexible measures" that can simultaneously play both input and output roles. Their models classify DMUs according to whether a flexible measure is behaving like an input or output. Namely, such dual-role flexible measures are still classified as inputs or outputs, and the status of input or output does not change for a particular DMU under evaluation. However, for targeted factors, it is desirable to change their values only up (or down) to target levels, and any deviations from the target levels are undesirable. Therefore, the status of the targeted factors changes from inputs to outputs, or vice versa. For example, a DMU only keeps the status of a performance measure as an output until that measure reaches a target level, and then the output status changes to input once that measure's value is larger than the target level. In a similar manner, an input can turn into an output once the input is decreased to a target level.

Taking into account the fact that lots of potential DEA applications exist with targeted factors being present, it is worthwhile to develop a DEA model in which such factors can be accommodated. Note that targeted factors can be treated under the ordinary production possibility set of conventional DEA models by simply transforming them into ones of regular type which basically represent their absolute deviations from targets; *i.e.*, we can treat these absolute deviations as regular inputs where "smaller-the-better" applies. While this deviation-based approach can be a technically equivalent alternative to the one developed in this paper, it is just an *ad hoc* approach and it is not based on an explicit production correspondence suited for targeted factors. Furthermore, the deviation-based approach can be used only under the assumption that deviations above and below targets are *equally* undesirable. For situations where this assumption cannot be made (the two types of deviations are not the same), an asymmetrical treatment of the production correspondence is required. To address these issues, we provide an axiomatic foundation to the deviation-based approach for ensuring its validity in this paper.

The remainder of the paper is organized as follows. In Section 2, the production possibility set is formulated by extending the standard DEA production possibility set under variable returns-to-scale (VRS) based on a set of properties postulated to suit the case of targeted factors being present. Section 3 is devoted to the development of three efficiency measures by extending the standard radial, slacks-based, and Nerlove–Luenberger measures. In Section 4, the proposed model and efficiency measures are illustrated by an application to the efficiency evaluation of 36 US universities. Concluding remarks are provided in Section 5.

2. Production possibility set with targeted factors

Assume that there are n DMUs and each DMU produces s regular outputs using m regular inputs. Formally, DMU j ($j = 1,2,\ldots,n$) uses a vector of inputs $x_j = (x_{1j},\ldots,x_{mj})^T \in \Re^m_+$ to produce a vector of outputs $y_j = (y_{1j},\ldots,y_{sj})^T \in \Re^s_+$. In addition, each DMU involves producing or using p targeted factors and each of the targeted factors has its own specific target level, denoted by $t_k \in \Re_+ (k = 1,\ldots,p)$, which is common across all DMUs (*i.e.*, the target level of each targeted factor is agreed upon by all DMUs). A vector of $z_j = (z_{1j},\ldots,z_{pj})^T \in \Re^p_+$ denotes the levels of DMU j's targeted factors, and a vector of $d_j = (z_{1j} - t_1,\ldots,z_{pj} - t_p)^T$ denotes the deviations of the levels of DMU j's targeted factors from the target levels. $X = (x_j)$, $Y = (y_j)$, and $Z = (z_j)$ denote the input, output, and targeted factor data matrices, respectively, where each column represents one of DMUs and each row represents the level of one of factors of the corresponding DMU. A function $I : R^p \to R^p$ is defined to represent the sign pattern of a given vector; for any vector $w \in R^p$, each element of $I(w) \in R^p$ assumes the value of 1, 0, or -1 depending on the sign of the corresponding element of w. To make our exposition simpler, we impose a regularity assumption on the data; $x_j \geqslant 0$, $y_j \geqslant 0$, $z_j \geqslant 0$, $x_j \neq 0$, $y_j \neq 0$, $z_j \neq 0$ $\forall j$, and $z_{kj} \neq t_k$ $\forall k, j$, but the latter assumption on z_{kj} can be easily relaxed.

Modifying the standard VRS technology set introduced by Banker et al. (1984), we postulate the properties of the production possibility set P as follows:

(A1) The observed activities (x_j, y_j, z_j) $(j = 1,\ldots,n)$ belong to P.
(A2) Given two activities $(x_{j_1}, y_{j_1}, z_{j_1})$ and $(x_{j_2}, y_{j_2}, z_{j_2})$ in P with $I(d_{j_1}) = I(d_{j_2})$, any convex combination of the two activities belongs to P.
(A3) If an activity (x, y, z) belongs to P, any semipositive activity $(\tilde{x}, \tilde{y}, \tilde{z})$ with $\tilde{x} \geqslant x$, $\tilde{y} \leqslant y$ and $|\tilde{z}_k - t_k| \geqslant |z_k - t_k|$, $\forall k$ is included in P.

Property (A1) is one of the basic assumptions typically made in any DEA models in the literature. Property (A2) is a modification of the standard convexity axiom. It assumes that the entire production possibility set consists of mutually exclusive subsets, and convexity condition is separately imposed on each of the subsets, not on the entire set. If we allow convexity on the entire set, it becomes possible that a convex combination of two DMUs results in a DMU dominating the two, which cannot be accepted in the usual production theory. (A2) prevents such situation while ensuring convexity of each subset.

Property (A3) is an extension of the standard free disposability axiom to accommodate targeted factors. Note that (A3) assumes that if an activity is feasible then another activity with a larger absolute deviation from the target, *ceteris paribus*, is also considered feasible. This property may or may not be relevant depending on the given evaluation context. If the property is postulated, the production possibility set becomes symmetric; otherwise, asymmetric. In what follows, we discuss each of the two cases.

2.1. Symmetric production possibility set with property (A3)

We now proceed to establish the production possibility set based on all of the three properties. First, we expand the set of the observed activities to effect (A3). Centered on $(x, y, z) = (0, 0, t)$, the factor space ($\subset R^{m+s+p}$) can be divided into 2^p orthants, where

Please cite this article in press as: Lim, S., Zhu, J. Incorporating performance measures with target levels in data envelopment analysis. European Journal of Operational Research (2013), http://dx.doi.org/10.1016/j.ejor.2013.04.051