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Invited Review

Constraints in container loading – A state-of-the-art review

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ABSTRACT

Container loading is a pivotal function for operating supply chains efficiently. Underperformance results in unnecessary costs (e.g. cost of additional containers to be shipped) and in an unsatisfactory customer service (e.g. violation of deadlines agreed to or set by clients). Thus, it is not surprising that container loading problems have been dealt with frequently in the operations research literature. It has been claimed though that the proposed approaches are of limited practical value since they do not pay enough attention to constraints encountered in practice.

In this paper, a review of the state-of-the-art in the field of container loading will be given. We will identify factors which – from a practical point of view – need to be considered when dealing with container loading problems and we will analyze whether and how these factors are represented in methods for the solution of such problems. Modeling approaches, as well as exact and heuristic algorithms will be reviewed. This will allow for assessing the practical relevance of the research which has been carried out in the field. We will also mention several issues which have not been dealt with satisfactorily so far and give an outlook on future research opportunities.

1. Introduction

More than 15 years ago, Bischoff and Ratcliff (1995a) argued, "… that existing approaches to container loading problems are each applicable only to a narrow part of the spectrum of situations encountered in practice …" (p. 377). They further claimed "… that a number of factors which are frequently of importance in practical situations have not received sufficient attention in the OR literature." (p. 378).

This paper is meant to be a review of the state-of-the-art in the field of container loading where we will pay special attention to the question whether and to which extent the factors mentioned by Bischoff and Ratcliff have been considered in the literature. This will allow for assessing the practical relevance of the research which has been carried out in the field.

In Section 3 we explain how the subject of our study has been delimited and what kind of literature has been included in our review. Basically, our investigation started by determining for each publication what problem type(s) has (have) been considered. Also, a brief (formal) statistical analysis of the respective data is presented in this section.

Section 4 represents the central part of our study. Departing from the paper by Bischoff and Ratcliff (1995a), a thorough analysis of the literature has been carried out in order to determine aspects relevant to container loading in practice. Furthermore, our study has been supplemented by aspects put forward in interviews with practitioners in the field. In general we found that these aspects are reflected by constraints. Thus, as one result of our investigation, we provide a comprehensive list of constraints practically relevant to container loading and – for the first time – introduce a scheme according to which they can be categorized. Furthermore, we pick up these constraint categories and describe in detail how the various approaches mentioned in the literature deal with the respective constraints.

In Section 5, we will summarize our observations concerning problem types and constraints considered in the container loading literature. Moreover, the state-of-the-art regarding different types of modeling approaches, as well as regarding exact and heuristic algorithms is briefly examined. Section 6 draws several general conclusions. In particular, we will mention several issues which – from our point of view – have not been dealt with satisfactorily so far and give an outlook on future research opportunities.

We start our presentation with a definition and a brief categorization of container loading problems.

2. Container loading problems – definition and categories

Container loading problems can be interpreted as geometric assignment problems, in which three-dimensional small items (called *cargo*) have to be assigned (packed into) to three-dimensional, rectangular (cubic) large objects (called *containers*) such that a given objective function is optimized and two basic geometric feasibility conditions hold, i.e.

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- all small items lie entirely within the container and
- the small items do not overlap.

A formal description of a solution to an assignment problem of this kind will be called a *loading pattern*.

We note that a large object might actually be a real container, but – according to the definition given – it could also be the loading space of a truck or a pallet which may be loaded up to a certain height.

In general, the small items may have any kind of regular (rectangular, spherical, etc.) or irregular shape. However, with very few exceptions, publications in the area of container loading deal with rectangular small items, only. As it is the general linguistic use in the literature, we will refer to these items as ''boxes''.

Furthermore we point out that by referring to the standard problems of cutting and packing (C&P) it is already implied that certain assumptions (e.g. concerning the objective function, the assortments of boxes and containers, etc.) hold for the container loading problems under discussion. In particular, whenever boxes have to be loaded, we will assume – in accordance with the existing literature – that only orthogonal placements are permitted, i.e. the surfaces of the boxes have to be aligned in parallel to the floor and the walls of the container.

According to the typology introduced by Wäscher et al. (2007), one can distinguish container loading problems with respect to the assortment of containers and the assortment of the small items. Containers (and likewise small items) belong to the same type if they are identical regarding their shape and dimensions. The assortment of containers (small items) is considered to be *weakly heterogeneous* if the containers (items) can be grouped into few classes, where each container type (item type) is available in relatively large quantities. It is called *strongly heterogeneous* if none or only very few containers (small items) are of identical shape and size.

Furthermore, one can distinguish between container loading problems, in which enough containers are available to accommodate all small items, and such problems, in which only a subset of the small items can be packed since the availability of the containers is limited. Different objectives are pursued in problems of these two categories. Problems of the first kind are of the *input (value) minimization* type, those of the second type represent the *output (value) maximization* type.

Input (value) minimization problem types are the following:

- Single Stock-Size Cutting Stock Problem (SSSCSP)
  Packing a weakly heterogeneous set of cargo into a minimum number of identical containers.
- Multiple Stock-Size Cutting Stock Problem (MSSCSP)
  Packing a weakly heterogeneous set of cargo into a weakly heterogeneous assortment of containers such that the value of the used containers is minimized.
- Residual Cutting Stock Problem (RCSP)
  Packing a weakly heterogeneous set of cargo into a strongly heterogeneous assortment of containers such that the value of the used containers is minimized.
- Single Bin-Size Bin Packing Problem (SBSBPP)
  Packing a strongly heterogeneous set of cargo into a minimum number of identical containers.
- Multiple Bin-Size Bin Packing Problem (MBSBPP)
  Packing a strongly heterogeneous set of cargo into a weakly heterogeneous assortment of containers such that the value of the used containers is minimized.
- Residual Bin Packing Problem (RBPP)
  Packing a strongly heterogeneous set of cargo into a strongly heterogeneous assortment of containers such that the value of the used containers is minimized.
- Open Dimension Problem (ODP)
  Packing a set of cargo into a single container with one or more variable dimensions such that the container volume is minimized.

As an extension to the typology of Wäscher et al. (2007), with respect to Open Dimension Problems (ODP) one may further differentiate between problems with a weakly heterogeneous assortment of cargo (ODP/W) and those with a strongly heterogeneous assortment (ODP/S).

The following output (value) maximization problem types can be distinguished:

- Identical Item Packing Problem (IIPP)
  Loading a single container with a maximum number of identical small items.
- Single Large Object Placement Problem (SLOPP)
  Loading a single container with a selection from a weakly heterogeneous set of cargo such that the value of the loaded items is maximized.
- Multiple Identical Large Object Placement Problem (MILOPP)
  Loading a set of identical containers with a selection from a weakly heterogeneous set of cargo such that the value of the loaded items is maximized.
- Multiple Heterogeneous Large Object Placement Problem (MHLOPP)
  Loading a (weakly or strongly) heterogeneous set of containers with a selection from a weakly heterogeneous set of cargo such that the value of the loaded items is maximized.
- Single Knapsack Problem (SKP)
  Loading a single container with a selection from a strongly heterogeneous set of cargo such that the value of the loaded items is maximized.
- Multiple Identical Knapsack Problem (MIKP)
  Loading a set of identical containers with a selection from a strongly heterogeneous set of cargo such that the value of the loaded items is maximized.
- Multiple Heterogeneous Knapsack Problem (MHKP)
  Loading a set of (weakly or strongly) heterogeneous containers with a selection from a strongly heterogeneous set of cargo such that the value of the loaded items is maximized.

We note that output (value) maximization is equivalent to the maximization of the container volume utilization if the value of the small items is proportional to their volume.

The following presentation will be based on this categorization. In particular, we will later analyze which of these problem types have actually been considered in the literature on container loading so far. We note that with respect to the definition of the terms ''cargo'' and ''container'' given above, container loading problems are considered here entirely as *three-dimensional* (3D) C&P problems. Obviously, we have already refrained from adding this adjective so far and we will continue to do so in the following.

3. Reviewed literature – characterization and basic analysis

For our review, we restrained our analysis to papers which are publicly available and have been published in English in international journals, edited volumes, and conference proceedings between 1980 and the end of 2011. Publications that were available online by the end of 2011 have also been considered.