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An economic order quantity model with a known price increase and partial backordering

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A constant unit purchase cost is one of the main assumptions in the classic Economic Order Quantity model. In practice, suppliers sometimes face a known price increase. In this paper, we develop EOQ models with a known price increase and partial backordering under two different assumptions about when the increase will occur. We prove the concavity of the extra profit functions for both scenarios if a special order is placed just before the price increases. A solution method is proposed and numerical examples are presented.

1. Introduction and literature review

When a supplier announces either a temporary reduction or a permanent increase in the unit purchasing cost of an item, the buyer can generally decrease his total purchasing cost by placing a larger-than-normal special order. We analyze the second case in this paper. In addition to the decision about whether and by how much to increase the order quantity, a problem that has been studied by a number of other researchers, we are including partial backordering of demand during stockout periods. The decision problem, then, is to determine, recognizing both the imminent price increase and partial backordering, the optimal quantity to order before a price increase.

This topic without partial backordering is investigated in several texts and articles on inventory control and management, including Naddor (1966), Brown (1967), Tersine (1967), Brown (1982), Tersine and Grasso (1978), Silver et al. (1988), Markowski (1986), and Gupta and Goel (1989). The problem was first formulated by Naddor (1966). Brown (1967) developed a model that appears to be different from Naddor's (1966). However, Brown (1982) has shown that there is no significant difference between the two proposed models. They both assumed that the buyer has an opportunity at the end of the current EOQ cycle to make a purchase at the current price and there will be a price increase for future orders. Otherwise, the usual assumptions of the basic economic order quantity model are made. Goyal and Bhatt (1988) assumed that n purchase orders of equal size are placed prior to an $(n+1)$st special order which is placed before or at time the price increase is to be effective.

Taylor and Bradley (1985) relaxed Naddor and Brown's timing assumption and assumed the price increase does not coincide with the end of a regular cycle. Lev and Soyster (1979) developed an EOQ model with a price increase and a finite planning horizon. Lev et al. (1981) studied an EOQ model in which one or more of the cost parameters or demand will change in future. They assumed that any change in the costs is likely to affect the demand rate. Kingsman and Boussofiane (1989) investigated an inventory control system in which the times between price increases follow a probability distribution function. Markowski (1990) investigated two different scenarios. The first one is the Special Order Strategy (SOS) in which a special order will be placed and, when the inventory again reaches zero, the buyer returns to an EOQ policy for all following orders. In the second one, which is the EOQ Strategy (EOQS), the buyer forgoes the special order and continues using an EOQ after modification for the price increase. He provided expressions for the actual total cost of both scenarios for any time of interest and showed that the choice of time horizon affects the choice of optimal strategy. Yanasse (1990) developed an EOQ model with an anticipated price increase in which, in order to determine the optimal order quantity, he used the criterion of minimizing the maximum error in terms of cumulative costs. Lev and Weiss (1990) developed a model which divides the planning horizon H into two parts including a closed interval $[0, T]$ and a half open interval $(T, H]$. For the first part the unit purchasing cost is C and for the second one it is increased to $C + C'$. Moreover, they assumed that the fixed ordering cost and unit holding cost are different in the two intervals. The aim of their research was to determine the numbers and sizes of orders during both finite periods. Erel

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(1992) investigated the effects of continuous changes in the unit purchasing cost and holding cost on the optimal order quantity and annual cost. Tersine (1996) developed an EPQ model with an announced price increase in which shortages are permitted and constant fraction of produced items will be defective. Abad (2006) used an EOQ model in a supply chain model including a producer, a vendor and an end user in which the producer considers both temporary reduction and increase in unit purchasing cost separately and the buyer places a special order in both situations. After a specific time the vendor will increase his unit selling price to the end user and the demand rate will be influenced by this decision. In Abad's model the special order quantity and increased selling price of the vendor are both decision variables. Huang et al. (2003) developed an EOQ model with an infinite planning horizon and a single announced price increase, with an option of placing a special order just before the price increases. They extended previous work in which it was assumed that the special order is an integral multiple of the new EOQ quantity. They assumed that there is a single instantaneous price increase and did not restrict the special order to be an integer multiple of the EOQ quantity. They used the Cesaro limit of a savings function to determine the optimal special order quantity. Then Lim and Rodrigues (2005) pointed out that their savings function was not Cesaro summable and their solution method was not correct. Shah (1998) developed a discrete-time stochastic inventory control model for perishable items when the vendor announces a price increase of units at some future time. In this model a constant fraction of on-hand inventory deteriorates and shortages are not permitted. Hsu and Yu (2011) studied an EOQ model with imperfect quality items under an announced price increase. They assumed that the defectives are screened out by a 100% inspection process and the defectives can be sold as a single batch at the end of the inspection process. Ghosh (2003) developed an EOQ model with full backordering in which an announced price increase is considered. Depending on the length of the intervening period between the announcement date and the effective date of price revision, two different models are proposed.

Our second addition to the basic EOQ model is the inclusion of partial backordering of demand during stockout periods. The first model for the basic EOQ with partial backordering was by Montgomery et al. (1973). Additional models that address the basic EOQ with partial backordering at a constant rate, which is the assumption we will use here, include Rosenberg (1979), Park (1982), Wee (1989), San José et al. (2005), Pentico and Drake (2009), and Taleizadeh et al. (2012, 2013). While these models differ in their choice of decision variables and, to some extent, the cost structures used, the only way in which their assumptions differ from those of the basic EOQ model is that they allow stockouts, with only a fraction of the demand during the stockout period being backordered. Many other authors have extended these models to include such considerations as a backordering rate that increases as the replenishment time gets closer, deteriorating inventory, and demand that varies depending on the selling price, the inventory level, or the passage of time. A comprehensive survey of this research may be found in Pentico and Drake (2011). In this paper we will use the decision variables and cost structure in Pentico and Drake (2009).

The paper most similar to the model we develop here is by Sharma (2009), which presents models for both a temporary price cut and a permanent price increase. In essence, he adds an imminent permanent price increase to the composite EPQ model in Sharma and Sadiwala (1997), which adds partial backordering at a constant rate and less-than-perfect-yield to the EPQ model. Sharma's model differs from ours in a number of ways, some of which are significant. When we discuss the assumptions of our model we will describe how they differ from those in Sharma (2009).

2. Problem definition and assumptions

Consider a situation in which a supplier announces that a price increase for an item will take place at or before a buyer's next scheduled ordering time. A logical response is to order additional units (place a special order) to take advantage of the lower (current) price prior to or at the regular replenishment time. Thus the manager must decide whether to place a special order and, if so, he must determine the size of the special order. If the price of an item will increase by an amount C' at a specific time, then the unit cost before that time will still be C and after that time will be $C_k = C + C'$. When the unit cost is C, the economic order quantity is:

$$Q^* = \sqrt{\frac{2AD}{iC}} \tag{1}$$

After the increase, the EOQ will decrease to:

$$Q_k = \sqrt{\frac{2AD}{i(C+C')}} = Q^*\sqrt{\frac{C}{C+C'}} \tag{2}$$

So the purchaser either orders a special quantity Q_S to take advantage of the lower price or ignores this opportunity and uses Q_k for all future orders. If a special order is to be placed, then the purchaser must determine the optimal value of Q_S, the size of the next order, after which all future orders will be of size Q_k.

Our assumptions are basically the same as those used in Taleizadeh et al. (2012), modified to reflect that the buyer is interested in taking advantage of the current lower price before it increases rather than buying at a sale price. In the following we discuss, where relevant, how they differ from those in Sharma (2009):

1. Shortages are allowed and a constant fraction β of the unsatisfied demand will be backordered.
2. There are different costs per unit for backorders, which is a cost per unit per period, and lost sales, which is a cost per unit and includes the lost profit on the lost sale. Sharma assumes that these two costs are the same and that they are both a cost per unit per period, so in his model there is a single cost for shortages based on the average shortage level.
3. All orders placed after time t_1 will be at the new, higher cost per unit.
4. Unlike Sharma, we assume that yield is 100%, so we ignore the costs of inspection. We also ignore the costs of in-coming transit, although those costs and inspection costs can be included in the fixed and variable costs of an order.
5. The order is paid for at the time of receipt, so we do not consider, as Sharma does, the holding cost of in-transit inventory.
6. The fixed ordering and unit backordering costs for both the regular and special orders are the same.
7. The holding cost per unit will increase after the price change to reflect the higher unit cost.
8. We assume that the unit selling price will not change, so the cost of a unit of lost sales will remain the same for the special order at the current price and will decrease to reflect the higher unit purchase price for future orders, which is not the case in Sharma's model since he does not recognize the lost sale cost as being different from the backordering cost.
9. Related to #6 and #8 is another significant difference between our model and Sharma's: the treatment of the maximum stockout and backorder levels for normal and special orders. Since we recognize that the cost of a lost sale is higher for a special order, due to the lower unit cost, than it is for a post-change order, we allow the maximum stock-