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Active Allocation of Systematic Risk and Control of Risk Sensitivity in Portfolio Optimization[☆]

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Abstract

Portfolio risk can be decomposed into two parts, the systematic risk and the nonsystematic risk. It is well known that the nonsystematic risk can be eliminated by diversification, while the systematic risk cannot. Thus, the portfolio risk, except for that of undiversified small portfolios, is always dominated by the systematic risk. In this paper, under the mean-variance framework, we propose a model for actively allocating the systematic risk in portfolio optimization, which can also be interpreted as a model of controlling risk sensitivity in portfolio selection. Although the resulting problem is, in general, a notorious non-convex quadratically constrained quadratic program, the problem formulation is of some special structures due to the features of the defined marginal systematic risk contribution and the way to model the systematic risk via a factor model. By exploiting such special problem characteristics, we design an efficient and globally convergent branch-and-bound solution algorithm, based on a second-order cone relaxation. While empirical study demonstrates that the proposed model is a preferred tool for active portfolio risk management, numerical experiments also show that the proposed solution method is more efficient when compared to the commercial software **BARON**.

Keywords: Branch-and-bound, systematic risk, risk sensitivity, factor model, second-order cone program.

1. Introduction

Modern portfolio selection theory was pioneered by Markowitz (1952) in his seminal mean-variance (**MV**) analysis. Although the downside risk measure, Value-at-Risk (see RiskMetrics (1996); Philippe (1996)), has recently become another standard industrial tool for risk management, the **MV** model remains one of the most popular tools in portfolio selection, especially in equity portfolio management. Three possible reasons may explain why this is the case: first, equity return is (almost) symmetrically distributed; second, the effect of diversification of risk can be well modeled by the variance term; finally, an intrinsic convenience lies in the computation of the **MV** model.

Decomposition of the entire risk according to the risk contributors is fundamental for portfolio risk management. RiskMetrics (1996) explicitly proposed the concept of marginal risk to measure the risk contribution of a given asset, which is defined as the difference between the risk of the entire portfolio and the risk of the portfolio without this asset. Actually, in the literature, except for the marginal risk given by

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