European Journal of Operational Research 228 (2013) 102–111

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Stochastics and Statistics

Pricing and risk management of interest rate swaps

Sovan Mitra [a,*], Paresh Date [b], Rogemar Mamon [c], I-Chieh Wang [d]

[a] Glasgow Caledonian University London Campus, London E1 6PX, United Kingdom
[b] Brunel University, Uxbridge, UB8 3PH, United Kingdom
[c] University of Western Ontario, London, Ontario N6A 5B7, Canada
[d] Meiho University, Pingtung, Taiwan

ARTICLE INFO

Article history:
Received 29 June 2012
Accepted 19 November 2012
Available online 12 December 2012

Keywords:
Swaps
Risk management
Financial mathematics
Numerical analysis
Stochastic interest rates

ABSTRACT

This paper reformulates the valuation of interest rate swaps, swap leg payments and swap risk measures, all under stochastic interest rates, as a problem of solving a system of linear equations with random perturbations. A sequence of uniform approximations which solves this system is developed and allows for fast and accurate computation. The proposed method provides a computationally efficient alternative to Monte Carlo based valuations and risk measurement of swaps. This is demonstrated by conducting numerical experiments and so our method provides a potentially important real-time application for analysis and calculation in markets.

1. Introduction

Financial derivatives pose many significant analytical and computational problems; for instance, see Popovic and Goldsman (2012), Albanese et al. (2012), Bhat and Kumar (2012), amongst others. Swaps are one of the most popular derivatives, which began trading in 1981 and are now a hundred billion dollar market (Toyoshima et al., 2011). A swap is a contract in which two counterparties exchange cashflows at prearranged dates, where the cashflow's value is derived from some underlying, e.g., interest rates, equities, exchange rates or commodities (Hull, 2000). Swaps have grown in popularity due to their ability to hedge out risks relating to the underlying. They also allow one to speculate upon the underlying over long periods of time (years) on a leveraged basis.

Interest rate swaps are one of the most important swaps that are traded (Azad et al., 2012). In addition to the risk management (Kim et al., 2012) and speculative uses, they can be used to manage interest rate borrowing costs, convert fixed borrowing costs to floating rate borrowing costs or vice versa. They have also become increasingly important since the global credit crunch to manage other financing costs (Ashton et al., 2012). Consequently, the necessity to accurately value and manage interest rate swaps is of increasing importance (Hsu and Wu, 2011).

Swaps have been researched with respect to general swap pricing and related issues. For instance there have been investigations into swap pricing under different asset classes, ranging from equity swaps (Dubil, 2012) to currency swaps (Tamakoshi and Hamori, 2013). Exotic swap pricing has also been investigated, such as variance swaps (Rujivan and Zhu, 2012) and swaptions (Bermin, 2012). Some specific swap types have been researched in more depth since the credit crunch began, for example credit default swaps. On credit default swaps there exists literature on their valuation in portfolios (Lin et al., 2011), the effects of sovereign credit risk upon their value (Ismailescu and Kazemi, 2010), their impact on credit spreads themselves (Chiarella et al., 2011) and pricing them using new techniques (Guarin et al., 2011).

In addition to general issues, swaps have been investigated in terms of their impact and interaction with particular factors. For instance, in Bhargava et al. (2012), the transmission of volatility and its impact on swaps in different markets is investigated. Chung and Chan (2010) explore the impact of credit spreads and monetary policy on swap spreads whilst Aizenman and Pasricha (2009) investigate the Federal Reserve's impact on swaps during the global financial crisis. Feldhutter and Lando (2008) analyse swap spreads and swap rates using a six factor model, and Huang et al. (2008) investigate the swap curve dynamics in US and Hong Kong dollar markets.

The research in interest rate modelling itself (such as in Schmidt (2011)) has become a well developed area in itself. Interest rate modelling began with simple stochastic differential equations representing interest rate changes and developing relations with respect to fixed income products. The interest rate modelling has been mainly divided into two areas. The first area focuses on models which there are models which are calibrated

* Corresponding author. Tel.: +44 203 369 3000.
E-mail address: sovan.mitra@gcu.ac.uk (S. Mitra).

0377-2217/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2012.11.032

by observing the term structure of interest rates, e.g. the Vasicek model (Chen and Hu, 2010). The second area encompasses interest rate models that are derived on a no arbitrage argument, such as a replicating portfolio. Examples of such no arbitrage models include the Hull–White model (Hull and White, 2009) and the Black–Derman–Toy model (Weissensteiner, 2010). Within both the no arbitrage and the term structure calibrated models, a major group of interest rate models are the one factor models (models with one source of randomness). The most significant one factor models include the Ho–Lee model (Ho and Lee, 1986) and the Cox–Ingersoll–Ross model (Zhou and Mamon, 2011). Multifactor models have also been developed and examples include the Brace–Gatarek–Musiela model (Suárez-Taboada and Vázquez, 2010) and the Heath–Jarrow–Morton model (Falini, 2010) to name a few.

The main areas researched within interest rate swaps include modelling with additional factors such as credit risk. For instance, Yang et al. (2010) price interest rate swaps taking into account bilateral risk of default, the impact of credit and liquidity risk is investigated by Liu et al. (2006) and Yang et al. (2010). Interest rates swaps have also been investigated with respect to macroeconomic factors such as in Azad et al. (2012) and with respect to different stochastic processes (Hsu and Wu, 2011). There has also been research on modelling different characteristics of interest rate swaps, such as the spreads (Fang et al., 2012; Huang and Chen, 2007; Chan et al., 2009) and the analyses of swap rates (Coleman and Karagedikli, 2012). However, there is no research on generic methods for interest rate swap valuation, nor methods offering fast computation of swaps and swap risk measures, either under deterministic or stochastic interest rates. This is despite that interest rates are known to be an important factor in finance to industry and regulators alike (Saha et al., 2009).

In this paper we introduce a new method to evaluate the fair price of swaps, the fixed interest rate and floating interest rate swap payments and interest rate swap risk measures. We introduce a new method that applies to a generic range of stochastic interest rate models and provides a method of fast computation of values and risk. We reduce the valuation and risk measurement (and management) problems under stochastic interest rates into one of solving a system of simultaneous linear equations with random coefficients. A method for solution to problems of this type was developed in Date et al. (2007), which is used here for obtaining accurate approximations.

It is worth noting that other authors working in the area of swaps have implemented approximate solutions to improve computation time and enable risk management (e.g., Plat and Pelsser (2009)). However this is not related to any work on interest rate swaps or risk measures. Also, computational methods in general are of interest to finance (e.g., Mitra and Date (2010)) propose a new computational method for calibration. Balbas et al. (2010) address the problem of optimising risk functions in finance. Hieber and Scherer (2010) propose an efficient Monte Carlo method of pricing barrier options and Deelstra et al. (2009) offer an approximation method for pricing basket options in short computation time.

The issue of valuation and risk management of interest rate swaps under stochastic interest rates is of importance to finance. The ability to incorporate stochastic processes into one's model is advantageous in finance but complicates modelling and analysis, such as in Fu and Yang (2012) and Bao et al. (2012). It can be empirically observed that interest rates fluctuate unpredictably over time and most interest rate models include a stochastic component. Furthermore, events such as the global credit crunch and the failure of Long Term Capital Management (Basu, 2011; Griffiths et al., 2010) demonstrate the importance of measuring and managing interest rate risks.

The plan of this paper is as follows. In Section 2 we introduce notation and formulate the valuation of swaps, fixed and floating leg payments in terms of a linear algebraic equation. In Section 3 we provide an approximate solution to these linear algebraic equations for swaps. In Section 4 we derive approximate solutions to swap risk measures under stochastic interest rates, including applying Generalised Extreme Value Theory. Finally we conduct numerical experiments to demonstrate the computational superiority of our method over the standard Monte Carlo simulation approach. We then end with a conclusion.

2. A linear algebraic formulation of swaps

In this section we formulate, under stochastic interest rates, the valuation of the swap, its scheduled fixed and floating rate payments in terms of a system of linear equations. The fixed or floating rate swap cash flow at each prearranged date is also known as the leg of the swap. The solution to the system of such equations will be discussed in Section 3.

2.1. Notation and preliminaries

In this paper we denote real vectors by boldface and matrices by capitalised letters. Let

- f_i denote a generic future cash flow at point in time t_i;
- p_i denote the swap running present value at time t_i of future cash flows w_j, where $j \geqslant i$;
- $\hat{p}_i$ denote the running present value at time t_i of future cash flows $\hat{w}_j$, where $j \geqslant i$ (the swap fixed leg's running present value at time t_i);
- r_i denote the one period interest rate (or short rate) during the time interval $[t_{i-1}, t_i]$;
- y_i denote the present value of generic future cash flows f_j where $j \geqslant i$;
- $\mathcal{V}(\cdot)$ denote variance;
- w_i denote the swap future cash flow at time t_i;
- $\hat{w}_i$ denote the swap fixed leg payment at time t_i (future cash flow);
- V_{FL} denote the present value now (time t_0) of all future swap floating leg payments;
- Υ denote the notional principal of the interest rate swap.

We assume that all our processes are defined under the probability space $\{\Omega, \mathcal{F}, \mathbb{P}\}$, where Ω denotes the sample space, the set of all possible events, $\mathcal{F}$ denotes a collection of subsets of Ω or events and $\mathbb{P}$ is the probability measure on $\mathcal{F}$ or events. The present value at time t_i of a \$1 cash flow occurring at time $t_N > t_i$ is given by

$$\frac{1}{\Pi_{j=1}^{N-1}(1+r_{j+1})}. \tag{1}$$

It is known that the absence of arbitrage in a financial market is equivalent to the existence of a risk-neutral (martingale) measure; in this paper we assume the dynamics of r_i is governed by a risk-neutral probability measure. We assume the short rate r_i is of the form

$$r_i = h(r_{i-1}) + z_i, \tag{2}$$

where $r_0 > 0$, $h(\cdot)$: $[0.1) \mapsto [0.1)$ is a known and deterministic function and $\mathbb{E}(z_i) = 0$, where $\mathbb{E}$ denotes the expectation operator. Eq. (2) is sufficiently general so that most single-factor short rate models will reduce to this form after discretisation. It is also assumed that z_i is defined on a time varying finite support such that $\mathbb{P}(z_i \in (-h(z_{i-1}), 1-h(z_{i-1}))) = 1$ holds at each time t_i. This