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Fuzzy turnover rate chance constraints portfolio model

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ABSTRACT

One concern of many investors is to own the assets which can be liquidated easily. Thus, in this paper, we incorporate portfolio liquidity in our proposed model. Liquidity is measured by an index called turnover rate. Since the return of an asset is uncertain, we present it as a trapezoidal fuzzy number and its turnover rate is measured by fuzzy credibility theory. The desired portfolio turnover rate is controlled through a fuzzy chance constraint. Furthermore, to manage the portfolios with asymmetric investment return, other than mean and variance, we also utilize the third central moment, the skewness of portfolio return. In fact, we propose a fuzzy portfolio mean–variance–skewness model with cardinality constraint which combines assets limitations with liquidity requirement. To solve the model, we also develop a hybrid algorithm which is the combination of cardinality constraint, genetic algorithm, and fuzzy simulation, called FCTPM.

1. Introduction

Through the introduction of the mean–variance model, Markowitz (1952) took a great step toward solving the portfolio optimization problem. Later, other researchers generalized this model by the expansion of assumptions and introduced new models, such as the mean absolute deviations model (Konno and Yamazaki, 1991), semi variance model, and semi absolute deviation model (Speranza, 1993). Some suggested that the mean–variance model can be applicable only for the problems with the symmetric investment return distribution, while, in the real world, this may not be true. The higher moments, rather than mean and variance (the first and second central moments), can be utilized to generalize the model for portfolios with asymmetric investment return distribution. "The basic intuition behind higher moment preferences is that investors are willing to give up some extra average return in exchange for lessening the chance of a big loss" (Loukeris et al., 2009). In fact, many investors prefer a portfolio with a large third order moment if the first and the second moments are equal (Simaan, 1997). Loukeris et al. (2009) used power utility function to utilize higher moments which indicates additional information of portfolios management. For the detail review of utility function in the literature, one may refer to Loukeris et al. (2009).

In this paper, we consider another important aspect of portfolio selection, called liquidity. The majority of short-term investors are willing to own the assets that can be sold easily. Furthermore, in many cases, stock price fluctuation of a company is caused by psychological and internal market factors and not necessarily by the company financial situation.

Asset liquidity was examined by Amihud and Mendelson (1986). Datar et al. (1998) introduced the turnover rate. Turnover rate, as a proxy for liquidity, is defined as the total amount of traded shares divided by the total net asset value (NAV) of the fund over a particular period. Since the turnover rate is the most important factor in portfolio liquidity (Marshalla and Young, 2003), this limitation is emphasized in the present paper. In our approach, the level of liquidity is measured by fuzzy credibility concept: see (Liu, 2009; Li and Liu, 2006). In our approach, turnover rate is controlled through fuzzy chance constraint, in order to select a portfolio with enough liquidity and a desired probability. This probability, as well as the level of liquidity, is determined by each investor. Chance constraint programming was introduced by Charnes and Cooper (1959). Fuzzy portfolio selection with chance constraint has been discussed with different assumptions and been solved by various methods: see for example (Huang, 2006, 2008; Li and Liu, 2008).

In order to manage the portfolio more efficiently, the number of assets in the portfolio should be limited. This limitation, called cardinality constraint, helps the investor to get less confused in the complex environments of the financial markets.

From another point of view, an asset with very low percentage in the portfolio increases the administration and supervisory cost. On the other hand, an asset with high percentage causes the portfolio risk to be increased. Thus, we impose an upper and lower percentage limit for each asset in the portfolio.

Generally speaking, we develop a mean–variance–skewness fuzzy portfolio model with cardinality constraint and by considering

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the fuzzy chance constraint to measure portfolio liquidity, called Fuzzy Cardinality Constraint and Turnover rate chance constraints Portfolio Model (FCTPM). Finally, to solve this model, we design a genetic algorithm based on cardinality constraint and by integrating fuzzy simulation.

The rest of this article is organized as follows. In Section 2, we present some basic concepts regarding the credibility theory and turnover rate. In Section 3, a fuzzy chance constraint mean–variance–skewness model is presented. In Section 4, an algorithm, based on fuzzy simulation combined with genetic algorithm, is developed to solve the model. Section 5 presents numerical examples to illustrate the proposed algorithm and its efficiency. In Section 6, sensitivity analysis with respect to variance and skewness is presented. Finally, in Section 7 we conclude the results and present some future research.

2. Basic concepts

2.1. Credibility theory

Since we use the credibility measure for evaluating portfolio turnover, we review its definition and some properties in this section.

Let ξ be a fuzzy variable with membership function μ, and u, r be two real numbers. Then, the credibility of ξ is defined as the average of the possibility and necessity of fuzzy event:

$$Cr\{\xi \leqslant r\} = \frac{1}{2}(Pos\{\xi \leqslant r\} + Nec\{\xi \leqslant r\}) \tag{1}$$

where possibility and necessity, two other major fuzzy measurements are defined as follows:

$$Pos\{\xi \leqslant r\} = \underset{u \leqslant r}{Sup}\, \mu(u) \tag{2}$$

$$Nec\{\xi \leqslant r\} = 1 - Pos\{\xi \succ r\} = 1 - \underset{u \succ r}{SUP}\, \mu(u) \tag{3}$$

2.2. Mean, variance, and the skewness of assets

Markowitz used mean and variance to evaluate an asset or a portfolio. Generally speaking, the expected value and variance of a portfolio represent the return and its risk, respectively.

Later, another criterion, skewness, was applied to evaluate an asset (or a portfolio) for other concepts of a portfolio. Mathematically speaking, mean, variance, and skewness of an asset are the first, second, and third central moment of that asset. In fact, since, in the real world, the return of portfolios is asymmetric, the concept of skewness was introduced to measure the asymmetry of fuzzy portfolio. Therefore, skewness measurement is used to control the high risk of the portfolios with asymmetric return.

By definition, the expected value, variance and the skewness of a fuzzy variable, ξ, with a finite expected value are defined as follows:

$$E(\xi) = \int_0^{+\infty} Cr\{\xi \geqslant r\}dr - \int_{-\infty}^{0} Cr\{\xi \leqslant r\}dr \tag{4}$$

$$V(\xi) = E[(\xi - E(\xi))^2] \tag{5}$$

$$S(\xi) = E[(\xi - E(\xi))^3] \tag{6}$$

2.3. Example: trapezoidal uncertain variables

Without loss of generality, we assume the return of an asset to be a trapezoidal fuzzy number with the following membership function:

$$\lambda(x) = \begin{cases} \frac{x-a}{(b-a)} & \text{if } a \leqslant x \leqslant b \\ 1 & \text{if } b \leqslant x \leqslant c \\ \frac{x-d}{(c-d)} & \text{if } c \leqslant x \leqslant d \end{cases} \tag{7}$$

where a, b, c and d are real numbers with $a \leqslant b \leqslant c \leqslant d$ and for simplicity it is represented by $\xi = (a,b,c,d)$.

Let assume $x_1, x_2, \ldots, x_n$ are n real numbers. By our notation, $f(x_1, x_2, \ldots, x_n)$ is defined as:

$$f(x_1, x_2, \ldots, x_n) = 4\theta^2 + 3\theta\psi + 4\psi^2 + 6\rho(2\theta + \psi + 2\rho) + \frac{(\psi - 2\rho)^3 + (\psi - 2\rho)^2|\psi - 2\rho|}{4(\theta + \psi)} \tag{8}$$

where

$$\theta = \sum_{i=1}^{n} x_i(b_i - c_i + d_i - a_i), \quad \psi = \sum_{i=1}^{n} x_i(b_i + c_i - d_i - a_i) \text{ and } \rho = \sum_{i=1}^{n} x_i(c_i - b_i) \tag{9}$$

where a_i, b_i, c_i, d_i, $i = 1, 2, \ldots, n$ are given real numbers.

Suppose the ith security return is presented as $\xi_i = (a_i, b_i, c_i, d_i)$, $i = 1, 2, \ldots, n$. Then, the expected value and variance can be converted into the following deterministic programming (Liu, 2009):

$$\sum_{i=1}^{n} x_i\xi_i = \left(\sum_{i=1}^{n} x_i a_i, \sum_{i=1}^{n} x_i b_i, \sum_{i=1}^{n} x_i c_i, \sum_{i=1}^{n} x_i d_i\right) \tag{10}$$

$$E\left[\sum_{i=1}^{n} x_i\xi_i\right] = \frac{1}{4}\sum_{i=1}^{n} x_i(a_i + b_i + c_i + d_i) \tag{11}$$

$$V\left[\sum_{i=1}^{n} x_i\xi_i\right] = \frac{1}{384} f(x_1, x_2, \ldots, x_n) \tag{12}$$

We also calculate the skewness of this portfolio as follows:

$$S\left[\sum_{i=1}^{n} x_i\xi_i\right] = \frac{\left(\sum_{i=1}^{n} x_i(d_i - c_i + b_i - a_i)\right)^2 \times \sum_{i=1}^{n} x_i(d_i - c_i - b_i + a_i)}{32} = \frac{-\theta^2\psi}{32} \tag{13}$$

2.4. Turnover volume

An important concern of many investors is to possess assets that can be liquidated easily. The liquidity of an asset can be identified by its turnover rate, defined as follows:

$$l = \frac{\text{the total volume of traded asset during a particular period}}{\text{the total net asset value (NAV) of the fund during a particular period}}$$

Furthermore, exchange market authorities and brokers benefit from high turnover rate, because they earn more fees by high traded asset volume, rather than by asset price change. That is why they are also willing to facilitate trading these assets more than other ones. Therefore, turnover rate is an important criterion for investors. High turnover rate of an asset is generally reflects its internal as well as its external events. Moreover, there is a direct relation between turnover volume and return on investment, see (Wermers, 2000; Brennan and Titman, 1994; Liu et al., 2003).

Due to the importance of asset liquidity and due to the preference of investors for the liquidity of their portfolio, we incorporate turnover criterion in our proposed model. We also define that the turnover rate of a portfolio consists of n assets as $\sum_{i=1}^{n} l_i x_i$, where x_i is the total capital invested in ith asset, and l_i is its turnover rate. A portfolio should be selected only if its turnover rate is not less than