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Decision Support

Robust multi-criteria ranking with additive value models and holistic pair-wise preference statements

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ABSTRACT

We consider a problem of ranking alternatives based on their deterministic performance evaluations on multiple criteria. We apply additive value theory and assume the Decision Maker's (DM) preferences to be representable with general additive monotone value functions. The DM provides indirect preference information in form of pair-wise comparisons of reference alternatives, and we use this to derive the set of compatible value functions. Then, this set is analyzed to describe (1) the possible and necessary preference relations, (2) probabilities of the possible relations, (3) ranges of ranks the alternatives may obtain, and (4) the distributions of these ranks. Our work combines previous results from Robust Ordinal Regression, Extreme Ranking Analysis and Stochastic Multicriteria Acceptability Analysis under a unified decision support framework. We show how the four different results complement each other, discuss extensions of the main proposal, and demonstrate practical use of the approach by considering a problem of ranking 20 European countries in terms of 4 criteria reflecting the quality of their universities.

1. Introduction

Inadvertent biases and uncertainties constitute an indispensable part of many decision support processes. They are related to the specification of a decision problem, the environment in which the decision has to be made, and the character of the value system and preferences of a Decision Maker (DM) [3]. The complexity of this issue has led to the development of a framework for robustness analysis, i.e. a theoretical basis and a diversity of dedicated multiple criteria decision support methods that take into account internal and external uncertainties observed in the actual decision situations.

As noted by Vincke [29], robustness is often used to formulate requirements with respect to decision processes, methods, solutions, or conclusions. In this paper, we are interested in investigating the robustness of the provided conclusions, i.e. whether they are valid for all or for the most plausible sets of model parameters. We focus on multiple criteria ranking problems with deterministic performance evaluations, and model the DM's preferences with additive multi-attribute value models [13] defined through holistic pair-wise preference statements (i.e. alternative *a* is (weakly) preferred over *b*).

The holistic judgments may require smaller cognitive effort from the DM in answering questions concerning her preferences than direct elicitation of the value function, e.g., through the bisection method [13]. However, there is typically more than a single value function compatible with the holistic statements. Obviously, the ranking of the alternatives can vary depending on the compatible value function used, and often the set of compatible functions must be reduced in size by introducing additional preference information to obtain a complete preorder or to determine the most attractive alternative.

Robust Ordinal Regression (ROR) [6,4] allows taking into account all instances of a preference model (in our case, the monotone additive value functions) compatible with the provided indirect preference information. These instances do not involve any arbitrary parametrization, so the whole space of compatible value functions can be explored. ROR methods provide the DM with two results, the necessary and possible preference relations for the set of considered alternatives. As far as methods designed for dealing with multiple criteria ranking problems are concerned, ROR has been implemented for the first time in UTA^{GMS} [6] that is a generalization of the UTA method [8]. Apart from considering all compatible value functions rather than just a single one, the UTA^{GMS} does not require the DM's ranking of reference alternatives to be complete and it assumes the use of marginal value functions that are general monotone, and not piece-wise linear. [9] extended the framework to consider all complete preorders compatible with

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the preference information and to determine the best and the worst ranks taken by each alternative.

A different way of handling multi-criteria problems having uncertain or imprecise values for the model was proposed in Stochastic Multicriteria Acceptability Analysis (SMAA). These methods apply simulation in order to provide the DM with indices describing the decision problem [23]; in particular, the original SMAA method [14] computes acceptability indices measuring the variety of different preferences that give each alternative the best rank, and SMAA-2 [16] extends it by introducing rank acceptability indices. They indicate the share of weights, criteria measurements, and other model parameters that assign an alternative to any rank from the best to the worst one. Ref. [18] proposed to derive pair-wise winning indices that indicate, for two alternatives, the probabilities of either being on a higher rank.

Both ROR and SMAA fail to consider some important issues. In particular, ROR methods analyse the sets of all, some, or no compatible instances of the preference model and the most and the least advantageous compatible model instances. However, in practical decision making situations, the necessary relation often leaves many pairs of alternatives incomparable, and it is desirable to answer how probable is it for an alternative to be preferred over another. Indication that an alternative could be ranked at its best or worst possible position with very high or extremely low shares of compatible preference models as well as knowing the most likely ranks an alternative can attain may change the preferred alternative of the DM similarly as risk attitude partially defines preference over risky outcomes in multi-attribute utility theory. Consequently, knowing the most and the least probable ranks and the probability of being preferred to another alternative may be valuable for practical decision support. In particular, a low probability of attaining a given rank indicates it to be sensitive for small changes in DM preferences.

SMAA-2 is traditionally applied with linear marginal value functions [27,28,24,19,1]. Such a limitation is arbitrary and restrictive, and it would be desirable for SMAA-2 to be applicable also with general monotone value functions. Furthermore, although SMAA-2 allows DMs to provide holistic preference judgments, they are used solely to derive linear constraints for the weights of the linear marginal value functions, and apart from the scaling, not to derive the piecewise linear functions themselves. Finally, although the rank acceptability indices of SMAA-2 can be estimated to within acceptable error bounds [25], they are not accurate. Therefore, an estimated rank acceptability or pair-wise winning index of 0 cannot be regarded with certainty, because they do not exclude the possibility of the alternative attaining a given position or being preferred over another alternative, respectively. Although the conditions under which such a situation is possible may be very specific, they are still consistent with the preference information provided by the DM. Thus, it is desirable to analyze estimations of the SMAA indices in the context of the necessary, possible, and extreme results of ROR and Extreme Ranking Analysis (ERA) to provide information on which particular outcomes occur with all, some, or no compatible preference models.

In this paper we overcome these shortcomings by combining ROR and SMAA in a joint approach. On the one hand, ROR is enriched by computing how probable are the possible relations. On the other hand, SMAA is extended by considering general instances of the preference model, admitting partial holistic judgments provided in an iterative manner, and confronting the indices estimated through Monte Carlo simulation with the results indicating necessary and possible preference relations and the corresponding extreme ranks.

The organization of the paper is the following. Section 2 presents the new combined approach for multiple criteria ranking problems. Section 3 considers extensions, discussing relations between the provided preference information and the outcomes of the combines approach, and introduces a procedure for selecting a representative value function. Section 4 provides an example application and Section 5 concludes.

2. The combined approach

We use the following notation:

- $A = \{a_1, \ldots, a_i, \ldots, a_n\}$ – a finite set of n alternatives;
- $A^R = \{a^*, b^*, \ldots\}$ – a finite set of reference alternatives on which the DM accepts to express preferences; we assume that $A^R \subseteq A$;
- $G = \{g_1, \ldots, g_j, \ldots, g_m\}$ – a finite set of m evaluation criteria, $g_j : A \to \mathbb{R}$;
- $X_j = \{g_j(a_i), a_i \in A\}$ – the set of deterministic evaluations on g_j; we assume, without loss of generality, that the greater $g_j(a_i)$, the more desirable is alternative a_i on criterion g_j;
- $x_j^1, \ldots, x_j^{n_j(A)}$ – the ordered values of X_j, $x_j^k < x_j^{k+1}$, $k = 1, \ldots, n_j(A) - 1$, where $n_j(A) = |X_j|$ and $n_j(A) \leqslant n$; consequently, $X = \prod_{j=1}^m X_j$ is the evaluation space.

The DM provides a partial preorder on the set of reference alternatives A^R, denoted by $\succsim$. In particular, the DM can state that a^* is at least as good as b^* ($a^* \succsim b^*$), a^* is indifferent to b^* ($a^* \sim b^*$), or a^* is strictly preferred to b^* ($a^* \succ b^*$). As a preference model, we use the additive value function:

$$U(a) = \sum_{j=1}^{m} u_j(a) \tag{1}$$

where the marginal value functions $u_j\left(x_j^k\right)$, $k = 1, \ldots, n_j(A)$ are monotone, non-decreasing and normalized so that the overall value (1) is bounded within the interval [0,1].

The pair-wise comparisons provided by the DM form the input data for the ordinal regression that finds the whole set of value functions being able to reconstruct these judgments. Such value functions are *compatible* with the preference information. Precisely, a set of general additive value functions $\mathcal{U}_{ROR}^{A^R}$ compatible with the provided pair-wise comparisons is defined with the following set of constraints:

$$\left.\begin{array}{l} U(a^*) \geqslant U(b^*) + \varepsilon, \quad \text{if } a^* \succ b^* \text{ for } a^*, b^* \in A^R, \\ U(a^*) = U(b^*), \quad \text{if } a^* \sim b^* \text{ for } a^*, b^* \in A^R, \\ U(a^*) \geqslant U(b^*), \quad \text{if } a^* \succsim b^* \text{ for } a^*, b^* \in A^R, \\ \left.\begin{array}{l} u_j\left(x_j^k\right) - u_j\left(x_j^{(k-1)}\right) \geqslant 0, \ k = 2, \ldots, n_j(A), \\ u_j\left(x_j^1\right) = 0, \ \sum_{j=1}^{m} u_j\left(x_j^{n_j(A)}\right) = 1, \end{array}\right\} E_{base}^{A^R} \end{array}\right\} E_{ROR}^{A^R}, \tag{2}$$

where ε is an arbitrarily small positive value. If $\varepsilon^* = max\,\varepsilon$, s.t. $E_{ROR}^{A^R}$ is greater than 0 and $E_{ROR}^{A^R}$ is feasible, the set of compatible value functions is non-empty. Otherwise, the provided preference information is inconsistent with the assumed preference model.

2.1. Necessary and possible preference relations

Robust Ordinal Regression applies all compatible value functions $\mathcal{U}_{ROR}^{A^R}$, and defines two binary relations in the set of all alternatives A [6]:

- Necessary weak preference relation, $\succsim^N$, that holds for a pair of alternatives $(a,b) \in A \times A$, in case $U(a) \geqslant U(b)$ for all compatible value functions;
- Possible weak preference relation, $\succsim^P$, that holds for a pair of alternatives $(a,b) \in A \times A$, in case $U(a) \geqslant U(b)$ for at least one compatible value function.