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# A lot-sizing and scheduling model for multi-stage flow lines with zero lead times

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ABSTRACT

We present a new model formulation for lot-sizing and scheduling of multi-stage flow lines which works without a fixed lead-time offset and still guarantees a feasible material flow. In the literature, multi-stage lot-sizing model formulations often use a fixed lead time offset of one period leading to increased planned lead times. Computational tests have shown that the total costs resulting from our new model formulation are at least 10% lower. Furthermore, we present a solution approach based on Fix-and-Relax and Fix-and-Optimize. Numerical results show that this solution approach generates high-quality solutions in moderate computational time.



## 1. Introduction

Great efforts are taken by industry to provide customers with reliable due dates while at the same time looking for ways to reduce throughput times. One such approach concerns Mathematical Programming models for lot-sizing and scheduling. These allow to generate capacity feasible plans and thereby to derive reliable due dates at minimum costs. However, in a multi-period, multi-stage model a fixed lead time offset of one period between adjacent production stages is usually modeled to secure a feasible material flow, resulting in unnecessary planned lead times. This paper will present a lot-sizing and scheduling model which works without a positive lead time offset, while still securing a feasible material flow at any point in time. Our objective function will minimize setup, holding, and backordering costs. Potential applications of our model are in the consumer goods or pharmaceutical industry where a few products are made to stock on a two- or three-stage flow line where the material flows between successive production stages are decoupled by inventories of intermediate products.

The paper is organized as follows: In Section 2 a literature review on simultaneous lot-sizing and scheduling models is presented followed by the assumptions underlying our models. Section 3 will start with a MIP model formulation for the multi-stage PLSP with a lead time offset of one (micro) period. This model formulation forms the basis for the model with no fixed lead time offset which concludes this section. Section 4 describes a combined Fix-and-Relax and Fix-and-Optimize approach which has been created to generate high quality solutions for decision problems of realistic size. Section 5 presents the test design and computational test results. Finally, Section 6 summarizes our findings and outlines three potential model extensions.

## 2. Literature review and problem statement

Models and solution procedures for simultaneous lot-sizing and scheduling have attracted many researchers in the last two decades starting with the pioneering work of Karmarkar and Schrage (1985). Since then, the focus of research has been on single machines. Note that a flow line with no intermediate storage may be treated – from a planning point of view – like a single machine. Subsequently, the term machine is used also as a collective term for a stage or any type of resource. In recent years research papers addressed the multi-machine as well as the parallel machine case. We will not review the vast number of proposals for single machines but concentrate on multi-level bills of materials on multiple machines. For the parallel machine case we refer to the literature review of Tempelmeier and Buschkühl (2008).

A major distinction of model formulations is the structure of the time axis. On the one hand we have big bucket models and on the other hand small bucket models. In big bucket models all items can be produced – at least in principle – within a period (i.e. a time bucket) on each machine. The number of periods to consider may remain small, however, there is little control of the sequence of items produced within a period unless further traveling salesman problem type constraints are added (like in the Capacitated Lot-sizing Problem with Sequence Dependent Setup Costs (CLSDs) approach (see Haase, 1996)). A further drawback is that most big bucket models require a lead time offset of one (macro) period in order to secure a feasible material flow between production stages

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while small bucket models usually only require a micro period as a fixed lead time offset. But even this micro period might be a day or a week. Note that for small bucket models the assumption is that at most one or two items may be produced per period. Hence, the number of periods to include into the model is relatively large and increases the computational effort.

As a compromise so called hybrid models have been proposed where macro periods of fixed length are associated with known external events (like external demands) while a number of micro-periods of variable lengths are within a macro period. In each micro period at most one single item may be produced. Here, a fixed lead time offset of one micro period will suffice to secure feasible material flows between stages. Which model is best cannot be answered in general and also depends on the algorithm proposed for its solution. Next, we will present some contributions for each time structure.

Model formulations based on the multi-stage CLSD have been considered by Haase (1996), Grünert (1998), and Sahling (2010). All these models assume a fixed lead time offset of one (macro) period. Sahling (2010) combines a standard MIP solver with a Fix-and-Optimize heuristic and is able to generate feasible solutions for all test instances with up to 40 items on 12 machines for a 17 periods time horizon. The solution quality is even better than a reference solution obtained by a MIP solver with a given CPU time limit. A model structure similar to the CLSD has been proposed by Mohammadi et al. (2009). Instead of TSP oriented sequencing constraints they assume that there must be precisely as many setups in each (macro) period as there are items on each machine. The setups do not need to be between different products. Machines as well as product structures are serial. They use a relaxed MIP model and generate solutions by a standard MIP solver within a rolling horizon setting. Problem sizes range from 3 to 7 end products on 3–7 machines over 3–10 (macro) periods. An attractive feature of their model is that a product $j$ produced on machine $m+1$ has to be finished *before* the start of production on the successor machine $m$ (i.e. a fixed lead time offset equivalent to the runtime of the lot on machine $m$).

In Haase (1994) proposed a small bucket model where at most two items may be produced within a period which he called the Proportional Lot-sizing and Scheduling Problem (PLSP). The PLSP is appealed for solving real-world single machine lot-sizing and scheduling problems because of its flexibility and exactness in modeling the timing of setup activities. For the single machine case it can be shown that the PLSP provides solutions that are as good as those generated by the (much more complex) continuous time models (e.g. Papageorgiou and Pantelides, 1996). The only requirement is that the length of a period does not exceed the (minimum) setup time plus the (minimum) time of producing a single batch (or a minimum lot size).

A PLSP-type model has been developed by Kimms (1999) for a multi-level bill of materials (BOMs) on multiple machines. He devised a genetic algorithm (GA) and concluded that for about 17.5% of the test instances no feasible solution could be generated and that the average deviation from the optimum was 19.9%. Hence, Kimms and Drexl (1998) conclude that even finding a feasible solution is a formidable task due to the additional restrictions caused by the BOM. Chang et al. (2004) adopted the model formulation of Kimms (1999) and introduced capacity allocation rules for products as well as upper bounds on lot sizes. As a solution procedure the authors also made use of a GA.

It seems that a GA constitutes an attractive meta-heuristic, because this has not only been chosen as a solution procedure by many researchers (e.g. Palaniappan and Jawahar, 2011) but also as a solution engine for production planning and scheduling in todays Advanced Planning Systems (e.g. SAP APO, see Stadtler and Kilger, 2008). The drawback of meta-heuristics, like the GA, is that a solution's quality (with respect to the optimal solution) is usually unknown. A specific feature of the model proposed by Palaniappan and Jawahar (2011) is that all the machines of the flow line have to be set up simultaneously if a new product is loaded to the flow line. The resulting non-linear integer programming model has also been solved by a standard solver for small test instances in order to have an indication of the solution quality of the GA.

To overcome model inadequacies a hybrid model named Generalized Lot-Sizing and Scheduling Problem (GLSP) seems advantageous. While the model by Fandel and Stammen-Hegener (2006) seems more conceptual than solvable by a standard MIP solver, Seanner and Meyr (2011) proposed an MIP model formulation including quantity-splitting. This means, one part of the production quantity can be directly consumed by the successors in the same micro period, while the second part is available with a lead time offset of one micro period. They experienced that generating feasible solutions for medium sized test instances (for 6–12 items, 3–9 machines over 3–8 (macro) periods) is still challenging even if embedded in a Fix-and-Relax heuristic.

A drawback of all the proposals mentioned above is that they require a lead time offset of at least one (variable micro) period. However, if there is no need in practice to have such a lead time offset, this planned lead time offset increases throughput times and intermediate inventories unnecessarily.

Last but not least we would like to point to a specific application area, namely a two stage production process (also known as make-and-pack production). Baumann and Trautmann (2010) devised a mixed integer programming model for make-and-pack production, which – for the first time – solved small real world decision problems to optimality in reasonable computational time with a standard MIP solver. However, in contrast to our model the batch size is fixed, not variable.

In summary, research is in a state that each proposal has its limiting assumptions and industry will have to choose the model formulation and solution approach carefully best suited for the decision problem at hand. However, model sizes which are solvable with reasonable numerical effort are still limited. Hence, our contribution is to present a new PLSP model formulation that is capable and flexible enough to generate high quality solutions for a great number of real world lot-sizing and scheduling problems within reasonable computational times if embedded into a Fix-and-Optimize heuristic.

The flow line model considered in this paper is based on a number of assumptions. First, we present those assumptions which also hold for the single-level, single-machine PLSP as described by Haase (1994) and later extended by Suerie (2005b):

- The planning interval is divided into discrete time periods (not necessarily of equal length).
- At most two items can be produced within a period.
- Production speed (on the machine) is constant within a period and may be product dependent.
- Lot sizes may be of any quantity.
- Primary demands only occur at the end of a period.
- Inventory holding costs are calculated based on the end-of-period inventory.
- A setup leads to setup costs and setup times.

Suerie (2005b) proposed a number of valuable extensions to the PLSP (in the context of a single machine) like campaign production, batch size restrictions and period overlapping setup times. The latter will also be incorporated in our model. Stadtler (2011) made suggestions regarding a PLSP model formulation for a multi-level bill of materials, single machine case. His model allows linear, convergent and divergent bill of materials. This also holds for the model proposed here.