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A methodology for probabilistic model-based prognosis

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ABSTRACT

This paper deals with the prognosis of complex systems using stochastic model-based techniques. Prognosis consists in this case in computing the distribution of the Remaining Useful Life (RUL) of the system conditionally to available information. In so doing, three main challenges arise from the industrial context. First, the model should unify the two classical approaches to describing complex systems: the bottom-up and the top-down approaches. The former uses elementary interacting components whilst the latter models the system's physical behavior by means of a set of differential equations. Second, the prognosis must integrate online information to provide a specific result for each system depending on their life events. Online information can take different forms (e.g. inspections, component faults, non detection or false alarm, noisy signal) which must all be considered. Third, the prognosis must supply ready, meaningful numerical results, the error of which must also be under control. This paper proposes a method addressing those challenges. The method is illustrated with two different examples: a simplified spring-mass system and a pneumatic valve for aeronautical application.

1. Introduction

In the aeronautical industry, health management and maintenance processes are among the main research topics for economical, ecological and industrial reasons (Inman et al., 2005; Vacthsevanos et al., 2006). However, the first challenge is to model industrial systems and their degradations. The variety and number of sources of uncertainty (e.g. forecast, complex system, unknown degradation process) encourage a probabilistic approach. System modelling can take various forms: macroscopic considerations (e.g. interacting components in or precise physical modelling (Daigle and Goebel, 2010), (Guan et al., 2009). This paper adopts a probabilistic method which considers both standpoints: the Piecewise Deterministic Markov Processes (PDMPs) introduced by Davis (1993) and studied by Jacobsen (2006) and Cocozza-Thivent (2011).

An interesting maintenance approach consists in using condition-based maintenance (CBM) to act on the system based on its current state and before its failure (Jardine et al., 2006). In the framework of control-limit decision rules, the CBM decision depends on an indicator associated with some thresholds (Jardine et al., 2006). It is often a degradation indicator as in Dieulle et al. (2003). Huynh et al. (2012) compare CBM strategies using degradation or age indicators.

A third indicator appeared recently: the Remaining Useful Life (RUL) of the system (Saxena et al., 2010; Vacthsevanos et al., 2006). It represents the remaining time before a failure occurs. Its definition in a CBM context remains unclear (see Jardine et al. (2006) for a partial definition). Moreover, in the literature, the RUL is computed using data only (Jardine et al., 2006, 2011), time-series forecasting (Yan et al., 2004), neural networks (Zemouri et al., 2003; Yu et al., 2006, 2012) or neuro-fuzzy systems (Wang et al., 2004) but never using models nor physics of failure. The RUL seems however of particular interest in this case, because unlike other indicators, it takes into account the future evolution of the system (Sikorska et al., 2011). After proposing an adapted definition of the RUL in such case, this paper focuses on the computation of this indicator.

The main contribution of this work is the proposition of a methodology to compute the RUL conditional on available observations of a system modeled with a PDMP. It consists of a two-step method. First, it requires the calculation of the conditional distribution of the system model states based on available information. Second, it involves the calculation of the reliability of the system with an initial state given by the previous law. Both steps are investigated, from theory to numerical implementation.

The remainder of this paper is organized as follows: Section 2 presents the mathematical problem associated with prognosis, and the PDMP model; Section 3 describes the two-step methodology to compute the RUL; Section 4 illustrates the method with two examples: one academic (spring-mass system) and the other aeronautical (pressure valve within the BLEED air system); Section 5

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concludes the discussion and enumerates different perspectives for future research.

2. System modelling and problem statement

2.1. Piecewise deterministic Markov process

Industrial systems are fundamentally multiphysics and can be considered at different scales, while their degradation processes are more or less known. A known degradation evolution often leads to a physical model (Vacthsevanos et al., 2006). When degradation process is unknown or when failure is totally random, one can use Poisson processes (Gaudoin and Ledoux, 2007) whereas a pure jump Markov process (Aven and Jensen, 1999) is well-suited to emphasize the system view of interacting components. For the industrialization of prognosis, one of the main challenges is to find a unified framework for all of those cases, and to develop adapted prognosis algorithms within this framework.

This purpose can be achieved through Piecewise Deterministic Markov Process (PDMP). PDMP's were first introduced by Davis (1993), and studied by Jacobsen (2006). They represent deterministic trajectories interspersed with random gaps. On the one hand, the deterministic motion allows us to consider continuous phenomena, which can either be the impact of some environmental stress factors (e.g. pressure, temperature) or the evolution of a physical degradation (e.g. crack propagation, corrosion). On the other hand, the gaps can model shocks or modifications in the degradation process, but they can also represent some changes in the state of other related components, or some maintenance tasks. Chiquet et al. (2009) use a PDMP to model crack growth. The deterministic motion represents the physical model of crack propagation, and the jumps allow us to consider shocks which influence the degradation. On the contrary, Lair et al. (2010) use a PDMP for system modelling, where the deterministic motion is the sole calendar time, and the jumps represent failures of components.

The PDMP's include non-homogeneous Poisson processes (even with covariates), pure jump Markov processes and deterministic models. The main modelling assumption within PDMP's is that randomness impacts the system only at discrete times. The PDMP's are irrelevant to describe "continuous" randomness, which should be considered through other processes such as Gamma processes (Castro et al., 2012; Huynh et al., 2012; van Noortwijk (2009)) which implies that degradation increases through permanent occurrences of very small increments.

In this paper, the definition of a PDMP refers to Cocozza-Thivent (2011), which is more general than Davis (1993) and Jacobsen (2006). One must define:

1. the deterministic motion, with a function ψ,
2. the jump process, denoted $(X_n,T_n)_{n\geqslant 0}$, where X_n represents the state of the system after the jump n, at time T_n.

The jump process must be a renewal Markov process (Jacobsen, 2006). It means that the law of the next jump $(X_{n+1},T_{n+1}-T_n)$ depends on the past $X_0,T_0,\ldots,X_n,T_n$ only through the value of the last position X_n. This law is given by $N(X_n,.,.)$, where $N=(N(x,dz,dt))$ is called the renewal Markov kernel of the process.

Let us consider a Polish space (E,ε), which represents the possible states of the system. A process Z with values in E is a PDMP if it can be written as follows:

$$Z(t)=\psi(X_n,t-T_n),\quad T_n\leqslant t<T_{n+1}, \tag{1}$$

with the following assumptions:

1. $\psi(x,t+s)=\psi(\psi(x,t),s)$ for all (s,t), and $s\to\psi(x,s)$ is right continuous with left hand limits $\forall x$;
2. $(X_n,T_n)_{n\geqslant 0}$ is a renewal Markov process, with $T_0=0$ by convention, and with kernel $N(x,dz,dt)=dF_x(t)Q(\psi(x,t),dz)$ such as:
   - dF_x is the probability function of $min(S_x,\alpha(x))$ with
     - S_x random variable with hazard rate $b(\psi(x,t))$,
     - $\alpha(x)\in\mathbb{R}_+$ deterministic time such as $\alpha(\psi(x,u))=\alpha(x)-u$
   - Q is a probability of transition on $E\times E$.

In this formula, the function dF_x represents the law of the time before the next jump from position x, and $Q(z,\cdot)$ represents the law of the position after a jump from position z. The assumptions on ψ are especially satisfied when ψ is the solution of an ordinary differential equation, as is the case for most of the physical models coming from mechanical considerations.

2.2. Problem statement

The degradation of the system is then modeled using a PDMP $Z=(Z_t)_{t\in\mathbb{R}_+}$ with values in a probability space (E,ε). The random variable Z_t represents the state of the system at time t. As usual with PDMP's (Davis, 1993), E is an hybrid space, i.e. $E=\mathbb{R}^d\times D$ with $d\geqslant 0$ and D a finite discrete space. This allows us to consider very general evolutions, with both continuous (e.g. temperature, crack growth) and discrete (e.g. on/off button, shock) phenomena, and different degradation modes.

The *useful domain* is modeled through a non empty subset $\mathscr{U}\subsetneq E$, corresponding to the authorized states of our system. Its complementary is denoted $\overline{\mathscr{U}}=E\setminus\mathscr{U}$. Thus $\mathscr{U}$ represents acceptable degradation states of the system, and $\overline{\mathscr{U}}$ the states to avoid. Because prognosis is related to maintenance applications and not safety constraints, $\overline{\mathscr{U}}$ may be different from the failure states of the system. The event $\{Z_t\in\overline{\mathscr{U}}\}$ means that the system could still work, but is not able to fulfill its requirements (fuel consumption, passengers entertainment, etc.) anymore.

For prognosis, the interesting quantity at time t is the remaining time before Z escapes from $\mathscr{U}$, i.e. the *remaining useful life* of the system:

$$\mathscr{Rul}_t=\inf\{s\geqslant t,\quad Z_s\notin\mathscr{U}\}-t \tag{2}$$

Since Z is a stochastic process, $\mathscr{Rul}_t$ is a random variable for each t. It is then characterized by its probability distribution function (PDF) $\mathscr{L}(\mathscr{Rul}_t)$. This function provides an idea of the remaining lifetime for the system under consideration. Besides, it includes typical quantities of interest (Jardine et al., 2006):

- the mean residual life with confidence bounds;
- the reliability until a time horizon;
- the time horizon corresponding to a risk of failure (a quantile of the PDF).

In order to provide a prognosis regarding a system specifically, it is essential to integrate the information on that particular system, available through monitoring or inspections.

The impact of information is illustrated in Fig. 1 with a one-dimension process. Scheduled preventive maintenance is based on a RUL curve from the stochastic model of the system without specific information, illustrated in Fig. 1a. Condition-based maintenance must consider the knowledge of the system state. Let us consider two different scenarios of observations. The first scenario (Fig. 1b) is an ideal case: one has a perfect knowledge of the past degradation of the system, of its actual state, and it evolves in auspicious conditions. Prognostic leads to the dashed curve result. The RUL with this information is more accurate than the previous one. The second case (Fig. 1c) is more realistic. The past observations (i.e. the triangles) are only partial in time and space, as well as