European Journal of Operational Research 225 (2013) 455–471

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Stochastics and Statistics

Dynamic journeying under uncertainty

Lauri Häme *, Harri Hakula

Department of Mathematics and Systems Analysis, Aalto University School of Science, P.O. Box 11100, FI-00076 Aalto, Finland

ARTICLE INFO

Article history:
Received 20 December 2011
Accepted 21 October 2012
Available online 2 November 2012

Keywords:
Stochastic processes
Transportation
Stochastic shortest path problem
Itinerary planning problem
Markov decision processes

ABSTRACT

We introduce a journey planning problem in multi-modal transportation networks under uncertainty. The goal is to find a journey, possibly involving transfers between different transport services, from a given origin to a given destination within a specified time horizon. Due to uncertainty in travel times, the arrival times of transport services at public transport stops are modeled as random variables. If a transfer between two services is rendered unsuccessful, the commuter has to reconsider the remaining path to the destination. The problem is modeled as a Markov decision process in which states are defined as paths in the transport network. The main contribution is a backward induction method that generates an optimal policy for traversing the public transport network in terms of maximizing the probability of reaching the destination in time. By assuming history independence and independence of successful transfers between services we obtain approximate methods for the same problem. Analysis and numerical experiments suggest that while solving the path dependent model requires the enumeration of all paths from the origin to the destination, the proposed approximations may be useful for practical purposes due to their computational simplicity. In addition to on-time arrival probability, we show how travel and overdue costs can be taken into account, making the model applicable to freight transportation problems.

1. Introduction

The urban itinerary planning problem involves determining a path, possibly involving transfers between different transport modes, from a specified origin to a similarly specified destination in a transport network. Common criteria used for evaluating itineraries include the total duration, number of transfers and cost (Androutsopoulos and Zografos, 2009; Pun-Cheng, 2012).

Passenger information systems provide real-time information on the status of transport services (buses, trams, trains, ferries, etc.) via mobile devices and displays at public transport stops. This makes it possible for a commuter to dynamically modify the planned journey in case of a delay or cancelation. For example, if a transfer from a transport service to another is unsuccessful due to a delay, the commuter may reconsider the remaining path to the destination.

We study a new objective for journey planning in scheduled public transport networks, motivated by uncertainty in travel times of transport services. Our goal is to maximize the reliability of a journey. In contrast to existing itinerary planning algorithms designed for scheduled public transport networks, where the path is a priori optimized with respect to an objective, for example, (Androutsopoulos and Zografos, 2009), our approach is to design the journey in a way that the probability of reaching the destination in time is maximized, even if some transfers are rendered unsuccessful in the course of time.

In the scheduling of an activity of random duration in general and in traveling under congested conditions in particular, the value of reliability is seen to be significant (Fosgerau and Karlström, 2010). Brownstone and Small (2005) and Small et al. (2005) argue that there is substantial heterogeneity in the valuation of reliability among motorists. In an empirical study on commuter behavior in California, where commuters chose between a free and a variably tolled route, the model in (Lam and Small, 2001) suggested that the average value of reliability is \$15.12 per hour for men and \$31.91 for women (in 1998 US dollars).

An example clarifying the main difference between stochastic and deterministic journey planning is shown in Fig. 1. The four nodes represent public transport stops (A,B,C,D) and the arrows between them represent scheduled public transport services ($1,\ldots,5$) operating within a specific time horizon. Each transport service has a specific schedule determined by the scheduled departure and arrival times shown next to the arrows.

(i) Let us first consider the deterministic case where the *realized* departure and arrival times of services are assumed to be equal to the *scheduled* departure and arrival times. For a commuter traveling from A to D, there are three feasible journeys: $(1,2)$, $(3,4)$ and $(3,5)$. In order to reach D as fast as possible, the commuter should follow the path $(1,2)$.

* Corresponding author. Tel.: +358 40 576 3585; fax: +358 9 470 23016.
E-mail addresses: Lauri.Hame@aalto.fi (L. Häme), Harri.Hakula@aalto.fi (H. Hakula).

0377-2217/\$ - see front matter
http://dx.doi.org/10.1016/j.ejor.2012.10.027

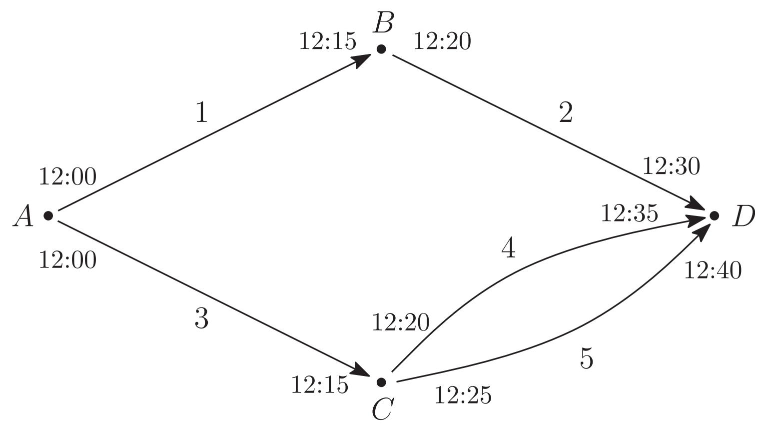

Fig. 1. The difference between stochastic and deterministic journey planning for a commuter traveling from A to D. The four points represent public transport stops (A,B,C,D) and the arrows between them represent public transport services $(1,\ldots,5)$. Initially, there are three possible journeys from A to D: $(1,2)$, $(3,4)$ and $(3,5)$. If the commuter initially chooses service 1, the success of the journey is dependent of the success of the transfer from 1 to 2 at stop B. If the commuter chooses service 3 first, the destination is reached if one of the transfers $3 \rightarrow 4$ or $3 \rightarrow 5$ is successful at stop C.

(ii) In the stochastic case, the realized departure and arrival times of services are not necessarily equal to the scheduled times. A transfer from a service to another may fail due to a delay, even if the transfer was feasible according to the deterministic schedule. For example, if the commuter initially chooses service 1, the success of the journey from A to D is dependent of the success of the transfer from 1 to 2 at stop B. Assuming that services 1 and 3 are equally likely to be delayed, it might be reasonable to initially choose service 3: There are more potential transfer options after leg 3.

More generally, in the model presented in this paper, a commuter wishes to travel from an origin node v_o to a destination node v_d within a time horizon $[0,T]$ using different transport services. Each transport service is represented as a sequence of *legs*. Each leg is associated with a *start node* and *end node*, as well as a random *start time* and a random *end time*. Adjacent nodes in the network are connected with similarly defined walking legs.

A path from the origin to the destination is represented as a sequence of legs, in which the start node of each leg is equal to the end node of the previous leg. We assume that at the end of a leg, the commuter receives information on which services have already visited the end node and which are yet to arrive. In other words, the customer "sees" the available successor legs of the current leg and may choose to (i) stay in the vehicle, (ii) transfer to another vehicle or (iii) get off the vehicle and start walking towards a nearby stop (or the destination). By using the start and end time distributions of legs, we define an optimal policy specifying the actions that are executed in different situations in order to maximize the probability of reaching the destination before T.

In addition to public transport, a similar journey planning problem arises in freight transportation by for-hire carriers. For this purpose, we show how travel and overdue costs can be incorporated in the model.

In summary, the problem can be characterized as a dynamic and stochastic path finding problem. Such problems are often modeled as *Markov decision processes* (Psaraftis and Tsitsiklis, 1993; Polychronopoulos and Tsitsiklis, 1996), in which the *actions* of a decision maker at a given *state* are independent of all previous actions and states. We first present a conditional Markov model, in which the path history is included in each state by defining states as sequences of legs in the transport network. That is, the current state is determined by the path taken so far. This model is further approximated by means of history independent models, in which the current state is defined as the current leg.

The remainder of this document is organized as follows: The journey planning problem under uncertainty is formalized in Sections 2, 3 and an algorithm that generates an optimal policy for the conditional Markov decision process is presented in Section 4. In Section 5, we approximate the conditional solution by assuming history independence and compare the solutions by analysis. The solution methods are evaluated by numerical experiments in Section 6.

1.1. Related work

There is a vast literature devoted to the deterministic path-finding problem in a transit network. Zografos and Androutsopoulos (2008) classify the approaches into the following types of formulations: (1) the headway-based model, in which a constant headway for each transit line is assumed (Wong and Tong, 1998) and (2) the schedule-based model, which assumes a fixed route and timetable for each transit line.

Our approach stems from model 2, for which most existing solution approaches are based on label correcting, label setting or branch-and bound, see for example (Zografos and Androutsopoulos, 2008; Peng and Huang, 2000; Modesti and Siomachen, 1998; Huang and Peng, 2001; Huang and Peng, 2002; Horn, 2003; Tong and Richardson, 1984; Tong and Wong, 1999; Ziliaskopoulos and Wardell, 2000; Ziliaskopoulos and Mahmassani, 1993; Brub et al., 2006; Cooke and Halsey, 1966; Cai et al., 1997; Chabini, 1998; Kostreva and Wiecek, 1993; Hamacher et al., 2006; Androutsopoulos and Zografos, 2009). Heuristic solutions, that are useful when the fast solution of the problem is essential, are presented in (Bander and White, 1991 and Tan et al., 2007).

In addition to the above-mentioned itinerary planning models, most of which are deterministic, our approach is closely related to the stochastic shortest path problem (SSPP). There are many different versions of the problem considered in the literature, each with a different objective for the optimal path (Murthy and Sarkar, 1997). Early studies related to the problem defined the optimal path to be the one that maximizes the decision maker's expected utility. This objective is motivated by the Von Neumann–Morgenstern approach of preference judgments under uncertainty (Loui, 1983). Bard and Bennett (1991) present heuristic methods involving Monte-Carlo simulation to solve the SSPP with a general non-increasing utility function. An exact algorithm for the SSPP with a quadratic utility function is presented in (Mirchandani and Soroush, 1985).

Recent studied objectives for the stochastic shortest path problem include (i) the maximization of the probability that the length of the path does not exceed a threshold value or finding the path