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Decision Support

An outranking-based general approach to solving group multi-objective optimization problems

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ABSTRACT

This paper presents a general approach to solving multi-objective programming problems with multiple decision makers. The proposal is based on optimizing a bi-objective measure of "collective satisfaction". Group satisfaction is understood as a reasonable balance between the strengths of an agreeing and an opposing coalition, considering also the number of decision makers not belonging to any of these coalitions. Accepting the vagueness of "collective satisfaction", even the vagueness of "person satisfaction", fuzzy outranking relations and other fuzzy logic models are used.

Our method transforms a group multi-objective optimization problem into a group choice problem on a decision set composed of a relatively small set of alternatives. This set contains the possible acceptable consensuses in the parameter space. Once such a set has been identified, other well-known techniques can be used to reach the final choice.

Main advantages: (a) Each individual decision maker is concerned with his/her own multi-objective optimization problem, only sharing decision variables; own constraints and own mapping between decision variables and objective space are allowed; (b) the search for the best agreement is not limited to portions of the Pareto frontiers; (c) no voting rule is used by the optimization algorithm; no to some extent arbitrary way of handling collective preferences is needed; (d) no assumptions of transitivity and comparability of preference relations are needed; and (e) the concepts of satisfaction/non-satisfaction do not depend on distance measures or other to some extent arbitrary norms.

Very good performance of the whole proposal is illustrated by a real-size example.

1. Introduction

Humans face the necessity to collaborate. To date, the methods for group decision aid strongly attract the attention of researchers in Operations Research, Management Science, and Social Science (e.g. Matsatsinis and Samaras, 2001; Costa et al., 2003; Frigotto and Rossi, 2011; Chuu, 2011; Fu and Yang, 2011; Morais and De Almeida, 2012; Liu et al., 2012).

Group decision-making is usually understood as the reduction of different individual preferences on a given set to a single collective preference (Jelassi et al., 1990). Unfortunately, classical Decision Theory is very limited in modeling group decisions, because the so-called collective preference is ill-defined. Group preference cannot be precisely defined and be free of paradoxes simultaneously. Phenomena such as non-transitivity and incomparability are quite often when group decision-making takes place. Arrow's Impossibility Theorem and Condorcet's Paradox make rationally doubtful every voting rule or way of aggregating ordinal preferences (Fernandez et al., 2010).

Usually, the group should make a choice on a relatively small set of actions, and many techniques have been proposed in this sense (e.g. Hwang and Lin, 1987; Jessup and Valacich, 1993). Many of them make some kind of aggregation of preferences in order to find "the best" collective choice (e.g. Leyva and Fernandez, 2003; Chiclana et al., 2007; Greco et al., 2011, 2012). Other techniques attempt to close the different systems of values and beliefs of the group members, making it easier to identify acceptable collective choices (e.g. Herrera et al., 1996). Besides, the group multi-objective decision problem on very large decision sets (characterized by constraints) has received comparatively very little attention.

This paper is devoted to multi-objective optimization problems (*MOPs*) solved by multiple decision makers (*DMs*), which is probably the most complex case of group decision-making. The action of maximizing a vector of objective functions $\boldsymbol{f}$ is ill-defined. In multi-objective programming for a single decision maker, the concept of best compromise solution is related to his/her system of preferences. From a normative point of view, assuming the existence of a value function $U(\boldsymbol{f})$ which agrees with the *DM*'s system of preferences, the "best" solution of a *MOP* should be obtained by maximizing U on a feasible

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region (Miettinen, 1999). Unfortunately, the practical value of this statement is strongly limited even for individual decision makers (see Roy, 1996 for a discussion of the practical limitations of decision actors).But there is no an identifiable system of preferences of a heterogeneous group of *DM*s. There is no a free-of-question model of collective preference for *MOP*s. There is no value function U for a group composed by members with heterogeneous value systems. There is no voting rule for constructing a group preference relation with desirable properties in order to solve group multi-objective optimization problems (*GMOP*s). A "priori" modeling of group preferences (considering the group as a single decision-maker) is only possible when a relatively homogeneous group is concerned, without strongly conflicting value systems; often, unanimity or a very strong majority is needed to set the decision model's parameters.

Some methods generate a representative Pareto sample, and then apply certain way of aggregating individual preferences in a model of collective preference, which is used to identify a final solution (e.g. Li and Hu, 2007; Hu et al., 2003). The most popular approaches propose a combination of group preference handling with interactive procedures of multi-objective optimization. (e.g. Lewis and Butler, 1993; Isermann, 1985; Wendell, 1980). In some methods the interaction is performed during the optimization process. In other methods the interaction is performed once a Pareto sample has been generated (e.g. Tapia and Murtagh, 1992). When successive solutions are replaced with others that seem to be better, most of these interactive methods implicitly assume that collective preference is transitive and comparable. Those interaction-based methods help to obtain acceptable agreements because each decision maker learns about the value systems of the others, and correspondingly fits his/her own. However, the final accepted solution may significantly differ from those that each decision maker would have considered satisfactory if the decision had depended solely on him/her. In this respect, the consensus does not result from the search in the set of possible solutions, but from mutual concessions. Group satisfaction is partial, because it is only achieved by knowing that no other result, more satisfactory to that, is possible. Group satisfaction depends on its member satisfaction. Each of them measures his/her own satisfaction by comparing the collective solution to that he/she would have chosen if it had been decided only by him/her, independently from the group.

This paper presents a fuzzy outranking-based generalized approach to solve *GMOP*s. Its main goal is to transform a group multi-objective optimization problem into a group choice problem on a decision set composed of a relatively small set of alternatives. This set contains the possible acceptable consensuses in the parameter space. Fuzzy outranking relations are used in order to create a computable model of satisfaction. Based on this model, measures of group satisfaction and non-satisfaction are optimized as a bi-objective problem, and its good compromise solutions are identified as possible consensuses. Once such a set has been identified, other well-known techniques can be used to reach the final choice.

The paper is structured as follows: Some background is given in Section 2. The problem description and our concept of the best solution to a *GMOP* are presented in Section 3. Our proposal is detailed in Section 4, mainly its axiomatic base (Section 4.1) and the way to find possible consensuses in the parameter space (Section 4.2). Section 4.3 discusses some ways to select the final agreement. The method is illustrated in Section 5 by solving a real-size example. Finally, we draw some conclusions in Section 6.

2. Some background

Following (Marakas, 1999; Fernandez et al., 2010), two basic structures for collective work oriented to decision making will be distinguished:

(1) Partner Association (Fig. 1a): It covers the case in which the whole group is responsible for the decision. There is symmetry between the different *DM*s, and the final decision is made according to previously established rules that define the way in which the group is "constituted" (group constitution). The symmetry does not imply that all group members have the same system of values or the same capacity to negotiate;

(2) Team or committee: It corresponds to situations in which symmetry is lost. There is a particular *DM* in charge of decision-making (we will refer to this entity as Supra-Decision Maker (*SDM*)), that makes his/her decision based on the collective, which processes a lower level of decision. Two situations can be distinguished: the members of this collective only interact with the *SDM* (team) (see Fig. 1b) or there is a complete interaction amongst all participants, but the main responsibility still falls on the *SDM* whose final judgment will be sustained by the best possible consensus of the members of the collective (committee) (see Fig. 1c) (Marakas, 1999).

3. Description of the problem

Most previous approaches to *GMOP*s assume that (a) group members share the objective functions and the same values on the scales of objectives; (b) group members agree with the constraint setting, but they have different goals and they assess different priorities to objective functions; and (c) group interaction is relatively easy. Under a very general view, we should also consider situations in which:

(i) The group members disagree with the constraint setting;
(ii) The group members have different values on the scales of objectives;
(iii) Different members can have different objectives;
(iv) Group interaction is difficult, even impossible.

Point (i) can be consequence of conflicting system of preferences. Point (ii) can arise from different beliefs (optimistic, pessimistic, etc.) even after group interaction. Point (iii) is extreme but possible mainly if there is no group interaction.

In formal way, let us consider a *GMOP* of the form

$$\textbf{Maximize}\ \boldsymbol{F}_j = (f_{1j}(z), \underset{z\in R_{F_j}}{f_{2j}(z), \ldots}, f_{Nj}(z)), \quad j = 1, \ldots M \tag{1}$$

in which

M is the number of group members; N is the dimension of the evaluation criterion space; $\boldsymbol{F}_j$ is the vector function being considered by the jth group member; z denotes a vector of decision variables; $f_{ij}(z)$ is the objective function that is associated with the ith evaluation point of view by the jth group member; R_{F_j} is a feasible region determined by a set of constraints that are imposed by the jth group member.

$f_{ij}(z) \neq f_{ik}(z)$ means that consequences of a decision z on the ith evaluation criterion may be not the same for two members.

A different mapping $\boldsymbol{F}_j(R_{F_j})$ corresponds to each group member. Such differentiation may be important when (i) some member imposes his/her own constraints; (ii) some member wants to consider additional objectives; (iii) although sharing the set of evaluation points of view, some members differ about the score $f_i(z)$ due to imprecision, uncertainty or scaling. In the rest of the paper O_j will denote the image of R_{F_j} by $\boldsymbol{F}_j$. The set $\cup_j R_{F_j}$ will be denoted by R_F.

3.1. What does solving Problem 1 mean?

For a given j, solving the problem