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Leading bureaucracies to the tipping point: An alternative model of multiple stable equilibrium levels of corruption

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ABSTRACT

We present a novel model of corruption dynamics in the form of a nonlinear optimal dynamic control problem. It has a tipping point, but one whose origins and character are distinct from that in the classic Schelling (1978) model. The decision maker choosing a level of corruption is the chief or some other kind of authority figure who presides over a bureaucracy whose state of corruption is influenced by the authority figure's actions, and whose state in turn influences the pay-off for the authority figure. The policy interpretation is somewhat more optimistic than in other tipping models, and there are some surprising implications, notably that reforming the bureaucracy may be of limited value if the bureaucracy takes its cues from a corrupt leader.

1. Introduction

OR has much to offer concerning complex societal problems (DeTombe, 2002), including parsimonious representations that concisely convey key dynamics, which is our objective here. There is a long tradition of and continuing interest in economic modeling of corruption (e.g., Rose-Ackerman, 2010). A recurrent theme is endogenous feedback or social interaction creating tipping points that separate multiple stable equilibria involving lower and higher levels of corruption. Multiple equilibrium models are appealing because they can explain two stylized facts without recourse to semi-tautological arguments about differences in culture or institutions, namely, there is (1) great heterogeneity across jurisdictions in the level of corruption and (2) stability over time in the level of corruption in any given jurisdiction (Dawid and Feichtinger, 1996; Andvig and Moene, 1990; Mishra, 2006).

Schelling (1978) offered what is perhaps the most famous such model, and thereby pioneered the idea of frequency-dependent equilibria in which individual incentives are a function of the aggregate level of corruption. There are other approaches. For example, Blackburn et al. (2006) model how corruption can harm economic development and low-levels of development can in turn promote greater corruption, and Mishra (2006) considers how corruption can develop via an evolutionary game. Lui (1986) uses an overlapping-generations approach to study the behavior of officials who maximize their expected payoff due to corruption. That paper considers the implications of multiple equilibria; however, its only dynamic aspect is young officials taking into account the expected payoff of bribes they might receive when they are old. We take Schelling's (1978) model as a point of departure both because it is so well known and because it was what inspired our thinking. In particular, we began by asking what a dynamic version of Schelling's model might look like.

The contribution of this paper is to suggest an alternative mechanism generating multiple equilibria, one which has somewhat different policy implications. Schelling's model considers the collective action of many small decision makers which feed back on these decision makers' private incentives. By marching in lock step they could shape system behavior. In contrast, we consider an "important" decision maker whose individual actions alone are sufficient to have macroeffects. We find threshold behavior and path dependency that looks similar to Schelling's model in its ability to explain great heterogeneity in corruption levels across societies at a given point of time, and persistence over time of both the lower-and higher-levels of corruption. We do not suggest that the mechanism described here is in any way better than others or

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even that the various mechanisms are mutually exclusive. Perhaps several mechanisms can play a role. Rather, we seek only to provide a concise description of this alternative mechanism.

The next section explains our model. The model takes the form of a linear-quadratic optimal dynamic control problem, so its qualitative solution structure can be derived analytically, as seen in Section 3. Section 4 concludes with the model's implications for a higher-level social planner or reformer who prefers for society to be in a low-corruption state. The social planner could be a constitutional convention designing the framework for a new system of government or an altruistic individual or agency that acts to monitor and respond to institutional corruption. In general, the present model offers somewhat greater optimism about the potential for a corrupt society to be pulled back to a low-corruption state.

2. The model

Schelling's model posits many decision makers who are essentially peers, each of whom rationally makes a binary choice about whether to be corrupt or not. In our model the masses are not so strategic; they just emulate norms set by high-level leadership. Rather, in our model there is just one individual whose decision calculus is modeled in detail, namely the head or chief executive of the organization (e.g., the head of state of a country). Furthermore that decision maker's choice is not binary (be corrupt or not) but continuous (how aggressively corrupt to be, e.g., how frequently one accepts bribes).

We refer to the decision maker as the "leader" not in a Stackelberg game theoretic sense but rather just in the ordinary sense of the word. We refer to the mass of people who take their cue concerning the acceptability of corruption from the leader as the "bureaucracy".

The leader can change his/her level of corruption instantaneously; it is a control variable, u. In contrast, the culture of corruption within the bureaucracy has a certain inertia, so it is represented by the state variable, x. Corruption grows under corrupt leadership and declines under a reformer in a manner we will describe shortly.

We have in mind incorporating and contrasting two particular corruption dynamics. The first is simply that the leader's own corrupt acts bring a direct benefit to the leader. The greater the degree of his or her own corruption, u, the greater is the benefit. This could be thought of as high-level or grand corruption.

However, the high-level leader does not accept petty bribes from everyday people directly. Rather, it is bureaucrats who extract bribes from the citizenry (e.g., to overlook infractions or to approve building or other licenses). Still, a corrupt leader will expect the bureaucrats to pass along a proportion of that bribe money. These payments could be thought of as a "franchise fee" or as "protection payments" purchasing protection from the enforcement powers vested in the leader's inner circle and entourage.

Hence, the leader's revenue from corruption has two terms, one that is driven just by u and another that is an increasing function of both u and the bureaucracy's total amount of corrupt revenue (proportional to x). The latter has an interaction that makes the cross partial derivative positive, so the function is not simply additive. The simplest function that captures this is to assume this 2nd component of the leader's corrupt revenue is proportional to the product of u and x.

Both the leader's own individual corruption and the bureaucracy's corruption are costly for the leader. Participating in corrupt practices directly (u) is costly because of the risk of being caught. Parameter β measures the difference between two effects, the leader's revenue that comes from bribes paid directly to the leader, not indirectly via the bureaucracy, minus the linear part of the cost of corruption (e.g., from enforcement risk). In a society whose institutions make it difficult for the leader to collect payoffs directly, the parameter β could be negative.

Presiding over a corrupt bureaucracy (x) is costly in terms of political popularity; the citizenry will blame the political leader if they are oppressed by pervasive extortion by government officials. Plausibly both costs are convex, and for simplicity we model them as being quadratic.

In order to avoid the problem of specifying salvage values after some finite term of office, we abstractly imagine the decision maker has an infinite time horizon but discounted at some (possibly fairly large) discount rate r, so the objective is

$$\max_u \int_0^\infty e^{-rt}\left(\alpha ux + \beta u - \frac{1}{2}u^2 - Cx - \frac{G}{2}x^2\right)dt.$$

All of the parameters are positive except perhaps β.

The state dynamics should reflect the idea that when the leader demands a large share of the bribe revenue, that will tend to increase corruption in the bureaucracy. This could be so for multiple reasons, including simple economic necessity (need to take more bribes to have enough money to pass along), practical factors (corrupt leaders have less incentive and ability to root out corrupt bureaucrats), and moral/sociological considerations (corrupt leaders signal a culture of permissiveness with respect to corruption). Conversely, if the leader is honest, the level of bureaucratic corruption will tend to decline, but not instantaneously. If we let δ denote the rate at which corruption ebbs under a completely honest regime, this suggests the degree of corruption in the bureaucracy might obey the simple dynamic:

$$\dot{x} = u - \delta x. \tag{1}$$

As a matter of realism and mathematical convenience, we presume there is a limit to how corrupt the leader can be, and scale that upper bound to 1.0. So we impose a control limit $u \leqslant 1.0$ which, given the state dynamics, also bounds the state variable. The control must be non-negative, for exogenous reasons, which via (1) implies that the state variable is also non-negative.

3. Solution

3.1. Analysis

We are considering a linear-quadratic infinite time nonlinear optimal control problem:

$$\max_u \int_0^\infty e^{-rt}\left(\alpha ux + \beta u - \frac{1}{2}u^2 - Cx - \frac{G}{2}x^2\right)dt,$$

subject to

$$\dot{x} = u - \delta x,$$
$$u \geqslant 0,$$
$$u \leqslant 1.0,$$

with x the state and u the control. The current value Hamiltonian is

$$H = \alpha ux + \beta u - \frac{1}{2}u^2 - Cx - \frac{G}{2}x^2 + \lambda(u - \delta x),$$

thus the costate equation is

$$\dot{\lambda} = (r + \delta)\lambda - \alpha u + C + Gx.$$

The necessary optimality condition for the control if no control constraints are active can be determined to be

$$H_u = \alpha x + \beta - u + \lambda = 0, \tag{2}$$