European Journal of Operational Research 225 (2013) 547–551

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Short Communication

Fast algorithms for scheduling with learning effects and time-dependent processing times on a single machine

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ARTICLE INFO

Article history:
Received 15 February 2012
Accepted 5 September 2012
Available online 14 September 2012

Keywords:
Scheduling
Learning/deterioration effect
Time-dependent
Due date assignment

ABSTRACT

We consider scheduling problems with learning/deterioration effects and time-dependent processing times on a single machine, with or without due date assignment considerations. By reducing them to a special assignment problem on product matrices, we solve all these problems in near-linear time. This improves the time complexity of previous algorithms for some scheduling problems and establishes the fast polynomial solvability for several other problems.

1. Introduction

In traditional scheduling problems, the processing times of jobs are fixed. This is based on the assumption that these times can be estimated with high precision in advance of processing the jobs. Since this assumption does not always hold, it also limits the applicability of these models. This is the reason why in recent years more and more researchers are investigating scheduling problems with variable processing times. Many different forms of variation have been studied, including learning effects, process deterioration, execution time-dependent processing times and processing times controllable by the scheduler.

As early as in the 1930s, Wright [44] noticed that in the aircraft industry the working costs per unit decreased with an increasing production output. He formulated the so-called 80% hypothesis, stating that with every redoubling of output the unit processing time decreases by 20%. This learning effect has significant impact in many mass production systems. For example, it was reported that in the semiconductor industry, the efficiency gains cause the price to drop by 10–30% [43]. A recent study of the application of learning-effect models in a new car-assembly plant was presented in [35].

Learning effects are important for production problems involving significant level of human activities, such as machine setup, machine cleaning, machine operating and controlling, machine maintenance, machine failure prevention, machine data reading/understanding/interpretation, and all kinds of manual work. They are especially important when the production environment changes. Such changes include, among others, new workers, investments in new machines or replacement of equipment, workflow changes, and the acceptance of new jobs.

Biskup [4] was the first to consider the learning effect in scheduling problems. He proposed the model $p_{[j]} = a_{[j]}j^c$, where $p_{[j]}$ and $a_{[j]}$, respectively, represent the actual processing time and "normal" processing time (i.e., without learning effect) of the job at the jth position in the schedule, and $c \leqslant 0$ represents the rate of learning. It is easy to verify that if $c = \log_2 0.8 = -0.322$, it corresponds to the aforementioned 80% hypothesis. Assuming the processing time of the first product is 100, then $p_{[k]} = 100k^{-0.322}$ describes the learning effect depending on the cumulative output. This effect is shown in Fig. 1, where the x-axis represents the number of products produced, and the y-axis represents the processing time needed.

If $c > 0$, then the model captures the effect of process deterioration [12]. Recently, Rustogi and Strusevich studied single machine scheduling problems with generalized positional deterioration effects and [28] investigated scheduling problems with the above learning effects on parallel machines and Mosheiov et al. [29,30] studied more general learning functions. A comprehensive survey about the learning effect in scheduling problems can be found in [5]. Further reviews of scheduling models with positional effects and learning were presented in [17,18,34].

On the other hand, in some other practical situations, the actual processing time of a job may depend on its starting time. For example, Gupta et al. [15] modeled a multiple loan repayment problem in financial management as this type of scheduling problem. In another example, steel ingots are to be heated to the required temperature before rolling can begin. The time taken to heat

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0377-2217/$ - see front matter
http://dx.doi.org/10.1016/j.ejor.2012.09.013

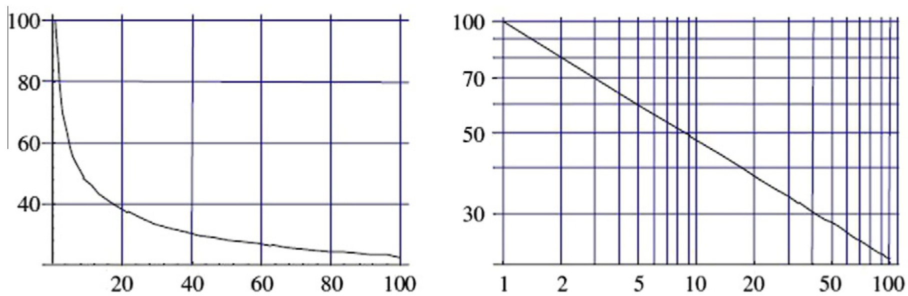

Fig. 1. The learning curve with 80% learning rate in a normal and a double-logarithmic coordinate system [5].

depends on the size and current temperature of the ingot, which further depends on the amount of time the ingot has been waiting to be processed. Similarly, in fire fighting, the time required to control a fire is longer if the start of the fire fighting effort is delayed [21,33]. Some other situations where such effects happen are: search for an object under worsening weather or growing darkness, use of medical procedures under deteriorating health conditions, repair of machines or vehicles under deteriorating mechanical conditions, hospital emergency ward scheduling, crime scene response scheduling for police, etc. [26].

Time-dependent processing times were first introduced by Browne and Yechiali [6], where they were developed to deal with the control of some queueing and communication systems in order to minimize cycle time of the server in the single-server cyclic-queue. They also discuss machine deterioration due to external shocks arriving according to a Poisson process. The most commonly used model is $p_{[j]} = a_{[j]} + bS_{[j]}$, where $a_{[j]}$ denotes again the "normal" processing time, b is a constant and $S_{[j]}$ denotes the starting time of the job in the jth position. If $b > 0$, then it reflects deteriorating processing time. These deteriorating effects may happen when the machine loses its efficiency when processing multiple jobs. It can also occur in the area of scheduling maintenance or cleaning assignment [25]. Mosheiov presented a V-shape policy and a $\wedge$-shape policy, respectively, in [25,26] for scheduling deteriorating jobs. Cheng et al. also presented a concise survey [7] for these problems.

The two types of variation described above are usually considered separately. Lee [22] was the first to use them simultaneously. Wang [38] (p. 4) proposed the following model, subsequently studied by Wang [40], Wang and Cheng [42,41] and Wang et al. [39].

$$p_{[j]} = (a_{[j]} + bS_{[j]})j^c, \tag{1}$$

where $p_{[j]}$, $S_{[j]}$, b and $c \leqslant 0$ all have the same meaning as before. This model reflects the following practical scenario: an operator obtains additional skills by learning from experience, at the same time, the machine that he/she operates is subjected to wear and tear, i.e., deteriorates with time [34]. Each of the above discussed variable processing times is a special case of this general model. If $b = 0$, we have the learning effect/deterioration problem; if $c = 0$, we have the case of time-dependent processing times.

The general problem we study in this paper may be stated as follows: n independent, non-preemptive jobs, $J = \{1,2,\ldots,n\}$, are available for processing at time zero and are to be processed on a single machine. A schedule is defined by a job sequence $\pi = ([1],[2],\ldots,[n])$, where $[j]$ represents the job that is in the jth position in π for $j = 1,2,\ldots,n$. Our objective is to determine a schedule which minimizes a general unified cost function that is the sum of various scheduling costs. Although we study a large number of unrelated scheduling problems, our analysis shows that they all have the common feature that the scheduling objective function can be expressed by using positional penalties. This unified cost function has the following simple formulation:

$$g(\pi) = \sum_{i=1}^{n} \xi_i \eta_{[i]}, \tag{2}$$

where ξ_i is a *positional, job-independent penalty* for *any* job scheduled in the ith position and $\eta_{[i]}$ is a job-dependent parameter for $i = 1,\ldots,n$. The model is a special case of the well-known linear assignment problem and it can be solved by sorting and matching the ith smallest η to the ith largest ξ. We'll refer to this as the *Sort-and-Match algorithm*. The optimality of this assignment follows from the well-known Hardy–Littlewood–Polya inequality/principle [16].

2. Applications of the sort-and-match algorithm in scheduling

Let $C_{[k]}$ be the completion time of the kth job in the processing sequence, whose processing time varies according to (1). The following crucial equality is proven by induction.

Lemma 1 38. *Assume that the jobs are processed from time zero and there is no idle time between them, then the completion time of the kth job in the processing sequence can be written as*

$$C_{[k]} = \sum_{j=1}^{k} \left(j^c \prod_{i=j+1}^{k} (1 + bi^c) \right) a_{[j]}, \text{ where } \prod_{i=k+1}^{k} (1 + bi^c) = 1 \text{ for any } k \text{ by definition.} \tag{3}$$

To simplify the notation, define

$$f_{jk} = j^c \prod_{i=j+1}^{k} (1 + bi^c) \text{ for } \quad j = 1,\ldots,n \; k = j,\ldots,n. \tag{4}$$

Then Eq. (3) above can be rewritten as

$$C_{[k]} = \sum_{j=1}^{k} f_{jk} a_{[j]}. \tag{5}$$

2.1. Minimizing earliness/tardiness with due date assignment

In this section, we show how our unified method can be used to solve a large set of scheduling problems involving due date assignment decisions. In such a flexible scheduling environment, the attainability of due dates and the feasibility of schedules are both taken into consideration at the same time. Taking advantage of this added flexibility may lead to improved overall system performance. Meeting due dates has always been one of the most important objectives in scheduling. While traditional scheduling models considered due dates as given by exogenous decisions (see [3]), in a more flexible and integrated system, they are determined by taking into account the system's ability to meet the quoted delivery dates. For this reason, numerous recent studies have viewed due date assignment as part of the scheduling process and showed how the ability to control due dates can be a major factor in improving