European Journal of Operational Research 223 (2012) 752–761

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Measurement of simultaneous scale and mix changes in inputs and outputs using DEA facets and RTS

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A R T I C L E I N F O

Article history:
Received 18 August 2011
Accepted 13 July 2012
Available online 23 July 2012

Keywords:
Data envelopment analysis
Changes in inputs and outputs
Returns to scale
Facets

A B S T R A C T

We show a new use of the efficient facets in DEA. Specifically, once we have identified all facets of the DEA technology, we are able to estimate the potential changes in some inputs and outputs, while fixing other inputs and outputs, ranges of simultaneous scale and mix changes in inputs and outputs, while proportionally increasing or decreasing other inputs and outputs, and, finally, the RTS. The proposed algorithms are applied to corporate planning processes of chemical companies.

1. Introduction

Data envelopment analysis (DEA) is an excellent method to evaluate the efficiency of decision making units (DMUs) which can decide their own management. However as Banker et al. (2004, p. 361) described, the focus of much of the DEA literature has been on ex-post facto analysis of already effected decisions rather than on the ante (planning) problem of how to use DEA knowledge in order to determine scale and mix of resources with other efficiency considerations. We herein present algorithms to answer the above types of questions. In addition, the proposed algorithms are applied to chemical companies and the use of the algorithms in corporate planning processes is demonstrated.

We refer to one of the flat sides of the efficient frontier, which is a linear combination of efficient DMUs, as a facet. This definition differs from that of a facet in convex polyhedron mathematics. Therefore, a facet of a DEA frontier is sometimes referred to as a DEA facet. In this paper, a DEA facet is simply referred to as a facet. In order to identify all facets and obtain their coordinates, combinatorial problems of efficient DMUs must be resolved. Thus, as Olesen and Petersen (2003) mentioned, obtaining all facets and their coordinates appears to consume huge computing power. However, all facets are identified from several hundred DMUs using specialized convex hull algorithms, such as *Qhull, cdd* and *lrs*, which Olesen and Petersen (2003) had used, and simple algorithms including an algorithm proposed by Amatatsu and Ueda (2011), whose algorithm determines all facets from 5000 DMUs with two inputs and two outputs within 700 seconds. Their algorithm is only one way of determining all facets in DEA. Other algorithms could be used in order to identify all facets as, for example, Olesen and Petersen (1996), Jahanshahloo et al. (2005, 2007) and Washio et al. (2011) have shown.

Few studies in the DEA literature have used the coordinates of facets. Closest distance algorithms are one such example. Fig. 1 illustrates the DEA algorithms using one input and one output. The efficiency of DMU*o* is measured. The efficient frontier is expressed by the bold line. The efficiency is measured by the distance between DMU*o* and the efficiency frontier in the hatched area. Although this distance can be measured using reverse convex algorithms, these algorithms require huge computing power for numerous DMUs, inputs, and outputs. Therefore, DEA algorithms usually measure the distance between a far point of the efficient frontier and the DMU*o*. In contrast, closest distance algorithms use the closest point. Briec and Lemaire (1999) used reverse convex programming to determine the closest point. Gonzalez and Alvarez (2001) measured the distance to the point with minimum inputs while maintaining outputs as the same quantity (isoquant). Takeda and Nisiho (2001) presented an algorithm in which a small circle is drawn around DMU*o* and the radius is gradually increased until the circle contacts the efficient frontier, as shown in Fig. 1. This radius is the distance (= efficiency score). Then, Silva Portela et al. (2003) used facets. They determined facets using *Qhull* and measured the distance between the facets and DMU*o*. They implicitly suggested that once all of the facets have been identified, the closest distance can be easily obtained. The closest distance algorithms, which adopt facets or hyperplanes, will be reviewed

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http://dx.doi.org/10.1016/j.ejor.2012.07.017

in Appendix B. The usual justification for using these algorithms is that closest reference DMU suggests a direction for improvement of inputs or outputs with less effort.

Olesen and Petersen (2003) outlined possible uses of the coordinates of facets in production possibility set. Based on their research, in this paper, we present a new use of the efficient facets in DEA. Specifically, once we have identified all of the facets of the DEA technology, we are able to estimate the potential changes in some inputs and outputs while fixing other inputs and outputs, the ranges of simultaneous scale and mix changes in inputs and outputs while proportionally increasing or decreasing other inputs and outputs, and, finally, the returns to scale (RTS).

The remainder of this paper is organized as follows. In Section 2, an output-oriented frontier, which envelops all observed DMUs, is proposed. The experimental computing times to find facets are also presented. Section 3 presents three algorithms. Section 3.1 presents an algorithm by which to measure the ranges of the simultaneous scale and mix changes in inputs or outputs, while fixing other inputs or outputs on the efficient frontier. Section 3.2 presents an algorithm by which to measure ranges of simultaneous scale and mix changes in inputs or outputs, while proportionally increasing or decreasing other inputs or outputs on the efficient frontier. Section 3.3 presents an algorithm by which to decide the RTS characteristics of facets according to the proposition of Banker and Thrall (1992). In Section 4, the use of these algorithms in corporate planning processes is outlined. Finally, conclusions are presented in Section 5.

The numerical examples and case studies draw upon financial data from 69 Japanese chemical companies, which are listed by the Nikkei Economic Electric Databank (NEEDS) (2010): (2010) and have sales of over 500 billion yen (approximately 6.5 billion US dollars). This data is listed in Appendix A. The inputs and outputs are as follows:

Inputs: fixed assets (Fixed A), number of employees (Emp) and current assets (Current A).
Outputs: sales (Sales), ordinary profit (Ordinary P), and net profit (Net P).

The coefficient of correlation of sales and ordinary profit is 0.45 in our data and is lower in many industries.

We also assume a virtual company called 'V Ltd.' which has financial indices of 275,183 million yen in fixed assets, 10,525 employees, 241,818 million yen in current assets, 580,000 million yen in sales, 22,000 million yen in ordinary profit, and 580,000 million yen in net profit.

2. Output-oriented frontier and experimental computing times to find facets

Let the number of inputs into DMUs be M, and let the number of outputs from DMUs be N. The coordinates in this $(M+N)$-dimensional space are denoted by $(\boldsymbol{x}, \boldsymbol{y})$. The efficiency of DMU$o$ is measured and the coordinates of DMUo are denoted by $(\boldsymbol{x}_o, \boldsymbol{y}_o)$.

We use the additive model. The non-oriented additive model is expressed as (Cooper et al., 2007):

$$\begin{aligned}
&\text{Object} \quad \max_{s^-,\boldsymbol{s}^+,\lambda}(\boldsymbol{e}\boldsymbol{s}^- + \boldsymbol{e}\boldsymbol{s}^+).\\
&\text{Subject to} \quad X\boldsymbol{\lambda} + \boldsymbol{s}^- = \boldsymbol{x}_o, \quad Y\boldsymbol{\lambda} - \boldsymbol{s}^+ = \boldsymbol{y}_o, \quad \boldsymbol{e}\lambda = 1, \qquad (1)\\
&\boldsymbol{\lambda} \geqslant 0, \quad \boldsymbol{s}^- \geqslant 0, \ \boldsymbol{s}^+ \geqslant 0, \quad \boldsymbol{e} = (1, 1, \cdots, 1).
\end{aligned}$$

The production possibility set (PPS) of linear program (1) is expressed as follows:

$$T = \{(X, Y) | X\boldsymbol{\lambda} \leqslant \boldsymbol{x}_o, Y\boldsymbol{\lambda} \geqslant \boldsymbol{y}_o, \boldsymbol{e}\boldsymbol{\lambda} = 1, \boldsymbol{\lambda} \geqslant 0, \boldsymbol{e} = (1, 1, \cdots, 1)\}. \qquad (2)$$

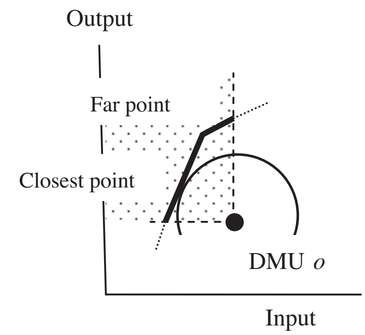

Fig. 1. Closest point.

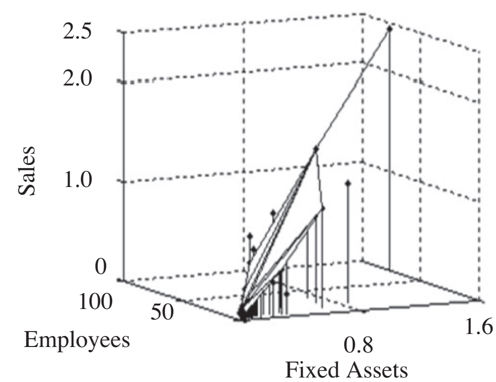

Fig. 2. Efficient frontier using linear program (1). (Unit; trillions of yen, 1000 employees).

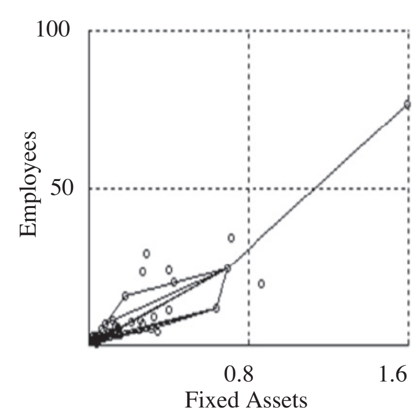

Fig. 3. Top view of the efficient frontier shown in Fig. 1.

Figs. 2 and 3 depict the efficient frontier from linear program (1) in three-dimensional space for two inputs of fixed assets and employees and one output of sales. In Fig. 2, the coordinates of fixed assets and employees are given on the horizontal plane, and the coordinates of sales are given on the vertical axis. One vertical line represents one DMU. Triangular planes and shaped lines on top of the vertical lines correspond to facets. These facets construct the efficient frontier. Fig. 3 shows a top view of the efficient frontier, where the dots represent DMUs. As shown in Figs. 2 and 3, the efficient frontier deduced from linear program (1) does not envelop all of the DMUs.

On the other hand, the output-oriented additive model is as follows: