European Journal of Operational Research 220 (2012) 777–785

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Ship-pack optimization in a two-echelon distribution system

Naijun Wen [a], Stephen C. Graves [b,c], Z. Justin Ren [c,d,*]

[a] Singapore–MIT Alliance, National University of Singapore, Singapore
[b] MIT Leaders for Global Operations Program, United States
[c] MIT Sloan School of Management, United States
[d] Boston University School of Management, United States

ARTICLE INFO

Article history:
Received 5 January 2011
Accepted 6 February 2012
Available online 25 February 2012

Keywords:
Ship-pack
Distribution system
Retail
Replenishment
Inventory optimization

ABSTRACT

In large distribution systems, distribution centers (DC) deliver some merchandize to their retail stores in size-specific packages, also called ship-packs. These ship-packs include cases (e.g., cartons containing 24 or 48 units), inners (packages of 6 or 8 units) or eaches (individual units). For each Stock Keeping Unit (SKU), a retailer can decide which of these ship-pack options to use when replenishing its retail stores. Working with a major US retailer, we have developed a cost model that balances DC handling costs, store handling costs and inventory-related costs at both the DC and the stores, and therefore can help to determine the optimum warehouse ship-pack for each SKU. We implement our model for a sample of 529 SKUs, and show that by changing ship-pack size for about 30 SKUs, the retailer can reduce its total cost by 0.3% - 0.4%. Interestingly, we find that most of the cost savings occurs at the DC level.

1. Introduction

There has been considerable research effort spent on optimizing inventory levels in a two-echelon distribution system (Hopp and Spearman, 1996). However, one important factor is often ignored: the choice of pack size that is to be shipped from the distribution center (DC) to the retail stores for a particular item (Wagner, 2002; van Zelst et al., 2007).

This research is motivated by such a real problem of choosing the right ship-pack quantity for a major US-based retailer (which we refer to as Beta hereafter). The ship-pack quantity can typically be one of three choices: an "each" or individual unit, an "inner" (a packaged set of eaches, on the order of 6–8 units), or a case (e.g., a box of 24 units). The DC incurs a greater handling cost when it replenishes with eaches or inners rather than full cases for two reasons. First, warehouse associates need to spend time cutting open cases so as to replenish the picking area for either inners or eaches. Second, each replenishment order from the store entails more work picking the packages. However, replenishing with cases could pose many problems for stores as well as DCs. First, the store inventory holding cost may increase since the order amount has to be a multiple of the case quantity, which could result in more store inventory. Second, this additional inventory may occasionally exceed the available shelf space at a store. When this happens, a store must put the extra units in a backroom or high-level shelf. This practice results in extra handling and additional labor cost, and can also increase the chances of pilferage and damage. Finally, the DC sees larger demand variability when stores are replenished in cases, and as a consequence, the DC has to carry more safety stock. Thus, it is of both the DC's and the stores' interest to find the optimal ship-pack that balances the DC handling cost, the store handling cost and the inventory-related costs at both the DC and the stores. This constitutes the main goal of this study.

In this research, we develop a cost model that can be used to evaluate and optimize the costs associated with a warehouse ship-pack in the two-echelon distribution system. Our cost model has the following contributions. First of all, it is store-specific. Currently, Beta uses an Excel model that is based on an EOQ formulation to determine the optimal ship-pack; this model calculates the cost at an "average" store, namely a store with the average demand rate, and thus, ignores the wide variation in demand rates across the retail stores served by a DC. We improve upon this model by developing a comprehensive model that generates ship-pack recommendations that account for the individual-store demand characteristics for all of the stores within the distribution system. Our model is also capable of including weekly forecasts over a planning horizon, say 26 or 52 weeks. Lastly, based on inputs from Beta, we include extra-handling cost at the store level that is absent in their current calculation. This extra-handling cost accounts for the labor required first to find storage space for items that cannot fit onto store shelves and then later to retrieve them. As far as we know, such a cost has never been considered in the literature on inventory replenishment. In sum, the contribution of this research lies

---

* Corresponding author at: Boston University School of Management, United States. Tel.: +1 6173839196.

E-mail addresses: naijun@gmail.com (N. Wen), sgraves@mit.edu (S.C. Graves), ren@bu.edu (Z. Justin Ren).

0377-2217/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2012.02.003

in the level of detail that we incorporate into the model, based on the business practices at Beta.

This paper is organized as follows. After literature review (Section 2), we introduce our research setting at Beta (Section 3). We then model the total cost in this system (Section 4), and with the data provided calculate the optimal ship-pack decisions for 529 SKUs, as well as total cost savings expected (Section 5). We then extend our model to consider the optimal inner-pack size choices (Section 6). Finally, we conclude (Section 7).

2. Literature

The economic order quantity (EOQ) problem is a century-old research topic that traces its root to a 1913 article by Ford Whitman Harris in *Factory: The Magazine of Management* (Erlenkotter, 1990). Today, the EOQ formula has become a pervasive textbook formula which every supply chain student has to learn. Traditional EOQ model assumes instant and infinite availability of products, deterministic and constant demand, constant fixed order cost and no shortages allowed (Hopp and Spearman, 1996). Three basic components are incorporated in the model: a fixed order or setup cost, a holding cost and a variable order or unit production cost. Later variations of the EOQ model have relaxed some of the assumptions. The Economic Production Lot size (EPL) model assumes a finite and fixed production rate; the Wagner–Whitin model relaxes the assumption on constant demand rate; and a variant of EOQ allows shortages and considers a back-order cost.

Although a great deal of academic literature exists on the EOQ model and its variants, very few studies have been done relating to pack size restrictions. Wagner (2002) acknowledges that the pack size could affect the order quantity in the real world. Silver et al. (1998) suggest a simple way of dealing with the pack sizes based on the form of the total cost curve in classical EOQ model. Since the total cost curve is convex, the best integral multiple of the pack sizes must be one of the two possible values surrounding the optimal continuous Q. However, a critical factor is ignored in the classical EOQ model: the handling cost of dealing with different case packs (including the individual unit which is essentially a case pack of one) both in the DC and the stores.

van Zelst et al. (2007) recognize shelf stacking process as the largest driver of the store operational cost. Moreover, the paper also demonstrates that the case pack size is the most important driver for stacking efficiency and concludes that increasing the case pack size could increase the stacking efficiency. However, Broekmeulen et al. (2007) later develop a regression model to show that high case pack sizes tend to cause shelf space shortages. Ordering behaviors from store managers are also significantly affected by the case pack size. The larger the case pack size for an SKU is, the more the store managers tend to deviate from system generated orders (van Donselaar et al., 2006). Thus, it is difficult to decide the best case pack size even at the store level.

Besides analysis that focuses on the impact of the case pack size on the retail level, some papers have extended such studies onto the DC level. A few papers show that pack size constraints could cause a bullwhip effect in the supply chain system, which consequently increases the total system cost (Geary et al., 2006; Lee et al., 1997a). This is in line with our modeling that larger ship-pack size induces larger demand variances at the DC level. Yan et al. (2009) address the problem of whether large case packs should be split prior to the retail level. They consider a two-echelon supply chain with a single distributor and multiple retailers under a periodic review inventory system. Assuming retail demand from an equicorrelated multivariate Poisson distribution, Yan et al. designed a factorial experiment with eight parameters including the number of retailers, the average retailer demand, heterogeneity of the retailer demand, the spatial correlation between retailer demands, the delivery pack size, the inventory safety factor, the review period at the retailer level and the critical protection period at the distributor level. Each parameter has three values that represent low, medium and high levels respectively. It is worth noting that the three pack sizes experimented are 1, 6 and 24, since these three pack sizes are also the most common among Beta's SKUs. Through simulation and ANOVA analysis, they find that of the eight parameters, the pack size has the most significant effect on amplifying demand variance up the supply chain, and it is also one of the most significant factors that result in larger stock-on-hand and back-orders at retailer level. Thus, the recommendation is to split packs at the distributor level. However, the paper ends on a cautionary note that soft costs such as breakage, pilferage and increased labor costs should be considered by management before any decision is made. It also suggests future research to include such financial implications, which is what this project does.

3. Research setting: a two-echelon distribution system with (*R*, *s*, *S*) policy

Beta is a major retail company with over 1500 stores in the United States that are supplied by a handful of regionally-located DC's. It carries approximately 12,000 SKUs. Each store is assigned to a DC; the SKUs carried by a store are replenished either from a DC or directly from the vendor (or supplier) by a flow-through policy. Under the flow-through policy, goods from the vendor are received at the DC and then directly sent to respective picking locations, from which store orders are fulfilled. Thus, the stores receive virtually everything from the DC.

As the choice of ship-pack quantity is made at each DC, we focus on a single two-echelon distribution system as depicted in Fig. 1. For Beta each DC serves between 200 and 400 stores.

Each store is replenished on a regular weekly schedule. Low volume stores are replenished once a week on a fixed day; higher volume stores are replenished two to five times a week, also on fixed days. Beta follows the R, s, S inventory control policy. At each review period R, the inventory control system checks the Inventory Position (IP) of all Stock Keeping Units (SKUs) at the store. If $IP \leqslant s$ (the reorder point ROP) for an SKU, then an order will be placed for that SKU to bring its inventory level to at least (the order-up-to-level OUTL).

4. Cost model

4.1. Notation and assumptions

Our goal is to develop a cost model that captures the relevant cost components affected by the ship-pack size for an SKU in the two-echelon distribution system. With such a cost model, we can

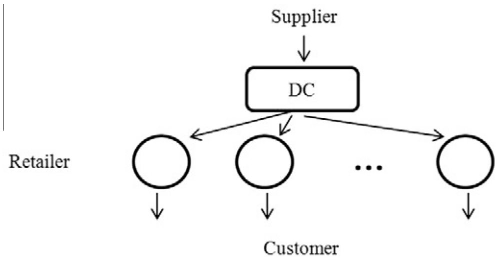

Fig. 1. Two-echelon distribution system with single warehouse and multiple retailers.