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Short Communication

An extension of partial backordering EOQ with correlated demand caused by cross-selling considering multiple minor items

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ABSTRACT

Zhang et al. (2011) proposed the partial backordering EOQ with correlated demand caused by cross-selling, where a portion of the sales of a minor item is associated with those of a major item. In this paper, we extend their model to make it more applicable to dealing with the inventory replenishment problem for multiple associated items. We formulate the model as a mixed integer nonlinear programming (MINLP) problem and develop a global optimum search procedure with the fill rate given. We further employ a one-dimensional search on the fill rate to obtain the minimum total inventory cost within a predetermined precision, which enjoys polynomial computational complexity.

1. Introduction

There has been much work regarding the deterministic EOQ with partial backordering and lost sales (Montgomery et al., 1973; Rosenberg, 1979; Park, 1982; Wee, 1989; Pentico and Drake, 2009; Pentico and Drake, 2011). Most of these studies assume no correlations in sales, so assumptions of independent demand and single items are usually implied in the models. However, it is generally recognized that demand for items is often correlated. A very frequent phenomenon is that some items are often purchased together by customers due to cross-selling effects (Anand et al., 1997; Kleinberg et al., 1998). As a result, demand for an item can be either promoted by successful sales or reduced by lost sales of its associated item (Brijs et al., 1999; Wang and Su, 2002), which results in the demand for an item being partially correlated with the sales of its related item. As a result, a joint inventory policy is highly applicable to the inventory management of associated items.

As regards the correlated demand caused by cross-selling, Zhang et al. (2011) proposed a two-item EOQ model, where the unmet demand for the major item can be partially backordered with lost sales that will reduce demand for the minor item. They developed an enumeration algorithm that can deal with only one minor associated item. However, in practice, there may be more than one minor item associated with a major item. For instance, the sales of a computer may affect the sales of expanded memories and hard disks simultaneously, and therefore the demand for the two minor items is partially correlated with that for the major item. Thus, if stockouts of the major item are permitted, a partial backordering EOQ model with multiple minor associated items should be considered.

In this paper, we extend the model in Zhang et al. (2011) to deal with the case of multiple minor items. For the solution, we first propose a global optimum search procedure on the order cycle of the major item (called *basic order cycle*) to yield the exact optimum with fill rate given. Then, we apply a one-dimensional search to the fill rate to obtain the minimum total inventory cost within a predetermined precision.

The rest of this paper is organized as follows. Section 2 formulates the model of the multi-item partial backordering EOQ with correlated demand caused by cross-selling. Section 3 addresses the optimal multipliers of minor items for each value of the basic order cycle with a given fill rate. Meanwhile, an upper and a lower bound on the optimal value of the basic order cycle are proposed. In Section 4, we develop a global search procedure to determine the optimal multipliers for different intervals of the major item's order cycle, based on which the inventory cost function can be optimized by a one-dimensional search on the fill rate. Section 5 provides a numerical example for illustration. The paper concludes in Section 6.

2. Model formulation

Suppose that there are n minor items for which demand is correlated to that for one major item that can be partially backordered. Because of cross-selling, sales of the major item may lead to additional demand for minor items. Thus, the minor

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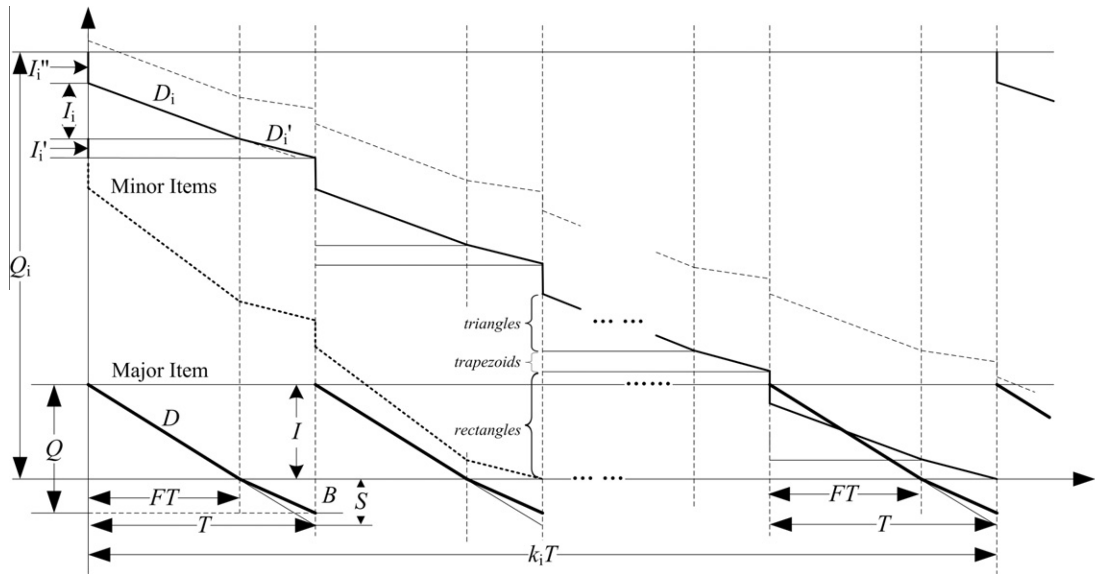

Fig. 1. Inventory levels of multiple associated items.

items can be either sold independently or promoted by joint sales with the major item, which means that demand for the minor items will decrease while the major item is stocked out. Following the assumptions in Zhang et al. (2011), the levels of inventories over the course of order cycles and their relationships to demand are shown in Fig. 1.

As pointed out in Zhang et al. (2011), the major item, say, a digital camera, is often more important to inventory management than the minor item, say, a memory stick. Therefore, the major item is often grouped into class A, which should be replenished more frequently to keep a lower inventory at a lower holding cost. This means that the order cycle of the major item is shorter than that of the minor item. In this case, it is practical to assume that the order cycle of minor item i is equal to a multiple k_i of the order cycle of the major item, where k_i is a positive integer (Silver et al., 1998; Hong and Kim, 2009). The order cycle of the major item is called the *basic order cycle* to distinguish it from that of the minor item.

For the major item, let T be the basic order cycle length, D the demand rate, and A the order cost. Let F be the fill rate, i.e., the percentage of the demand that is filled from the shelf stock. Let C_o be the opportunity cost per unit of lost sale, C_h the inventory cost of holding one unit item for one unit of time, and C_b the cost of keeping one unit backordered for one unit of time. Let β be the backordering rate, i.e., the fraction of stockouts that will be backordered. Denoting the maximum stockout level during a basic order cycle as S and the maximum backorder level as B, we have $B = \beta S = \beta D(1-F)T$.

The notations for minor item i are:

D_i	the demand rate for minor item i while the major item is not stocked out, in units/unit time
A_i	the order cost, in \$/order
C_{oi}	the opportunity cost per unit of lost sales, including lost profit and any goodwill loss, in \$/unit
C_{hi}	the unit holding cost for one unit of time, in \$/unit/unit time
λ_i	lost sales of minor item i caused by one unit of lost sales of the major item, in units/unit
D_i'	the demand rate for minor item i while the major item is stocked out, where $D_i' = D_i - \lambda_i D$

The total inventory cost of the major item is given by Eq. (1) (Pentico and Drake, 2009; Zhang et al., 2011).

$$\Gamma(T,F) = \frac{A}{T} + \frac{C_h DTF^2}{2} + \frac{\beta C_b DT(1-F)^2}{2} + C_o D(1-\beta)(1-F). \tag{1}$$

For minor items, let Δ_i represent the time-weighted inventory of minor item i. This can be calculated by summing up the areas of all triangles, trapezoids, and rectangles (see Fig. 1) under the inventory level line, which yields

$$\Delta_i = k_i \cdot \frac{1}{2} FT \cdot I_i + k_i \cdot \frac{1}{2}(FT+T) \cdot I_i' + \sum_{t=1}^{k_i} (t-1) \cdot (I_i + I_i' + I_i'') \cdot T.$$

The total inventory cost of minor item i is equal to the sum of the order cost, the inventory holding cost, and the opportunity cost of lost sales. Note that the lost sales of minor item i caused by the lost sales of the major item in one unit of time is $\lambda_i D(1-\beta)(1-F)$. Thus, the total inventory cost of minor item i per unit of time is

$$\Gamma_i(k_i,T,F) = \frac{A_i}{k_i T} + \frac{\Delta_i C_{\mathrm{hi}}}{k_i T} + C_{\mathrm{oi}} \lambda_i D(1-\beta)(1-F).$$

Substituting $I_i = D_i FT$, $I_i' = D_i'(1-F)T = (D_i - \lambda_i D)(1-F)T$, and $I_i'' = \lambda_i B = \lambda_i \beta D(1-F)T$ into the above equation and merging similar terms, we have

$$\begin{aligned}\Gamma_i(k_i,T,F) = {} & \frac{A_i/k_i}{T} + \frac{\lambda_i C_{\mathrm{hi}}}{2} DTF^2 + \lambda_i C_{\mathrm{oi}} D(1-\beta)(1-F) \\ & + \frac{\lambda_i(1-\beta)(k_i-1)C_{\mathrm{hi}}}{2} DTF \\ & + \frac{[(d_i-\lambda_i)k_i + \lambda_i\beta(k_i-1)]C_{\mathrm{hi}}}{2} DT,\end{aligned} \tag{2}$$

where $d_i = D_i/D$ and $d_i > \lambda_i$. The decision model minimizing the total inventory cost of all the items is formulated as

$$\begin{aligned}\min_{\mathbf{K},T,F} \Gamma(\mathbf{K},T,F) &= \Gamma(T,F) + \sum_{i=1}^{n} \Gamma_i(k_i,T,F) \\ &= \frac{1}{T}\left(A + \sum_{i=1}^{n} A_i/k_i\right) + \frac{T}{2}\left(\sum_{i=1}^{n} h_i k_i + h\right) + C_F\end{aligned}$$

$$\text{s.t.} \tag{3}$$

$$\begin{cases}\mathbf{K} = (k_1, k_2, \ldots, k_n)^T \\ k_i \in \{1,2,3,\ldots\} \\ 0 \leqslant F \leqslant 1,\end{cases}$$