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An analytical coupled homotopy-variational approach for solving strongly nonlinear differential equation

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ABSTRACT

In the present paper, a novel technique combining the homotopy concept with variational formula has been presented to find accurate analytical solution for nonlinear differential equation with inertia and static non-linearity. The obtained results are compared with other analytical and exact solutions to confirm the excellent accuracy and correctness of the approximate analytical technique. The results of the present paper are valid for large amplitudes of oscillation; also the approximate solutions give excellent result than other methods. We concluded that the first order approximation obtained in current work are almost the same with exact solutions, also works very well for the whole range of initial amplitudes.

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1. Introduction

With the fast development of research in nonlinear science, there appears ever increasing interest of scientists and researchers in novel techniques to find efficient methods for obtaining approximate solutions to nonlinear differential equations. The study of given nonlinear oscillator problems is of crucial importance in many branches of sciences such as mechanics, engineering and applied Mathematics. During the recent years, a large variety of new analytical approximate techniques to solve nonlinear differential equations include Energy Balance Method (EBM) [1,2], Amplitude-Frequency Formulation (AFF) [3,4], Variational Iteration Method (VIM) [5], Homotopy Perturbation Method (HPM) [6,7], Homotopy Analysis Method (HAM) [8], Iteration Perturbation Method (IPM) [9,10], Min-Max Approach (MMA) [11], Laplace Transform (LT) [12], Hamiltonian Approach (HA) [13–15], Variational Approach (VA) [16,17], Global Residue Harmonic Balance Method (GRHBM) [18,19], Coupled Homotopy-Variational Approach (CHVA) [20–25] are most accepted approximate methods in studying non-linear models arising in physics, mathematics and engineering that are constantly being developed or applied to more complex non-linear systems.

In recent decades, some researchers have studied the behavior of the current differential equation, for example. Abd El-Latif [26] proposed a new approach by combining the linearization of the governing problem with the (HBM), Molla et al. [27] used the (HBM) to obtain accurate approximate analytical higher-order solutions for nonlinear problem.

In current work, a novel and different technique called the coupled of homotopy with variational approach [5,6] has been employed to find accurate periodic solutions to nonlinear oscillators. The coupled method is applied to derive highly accurate analytical expressions for approximate formulas of frequency.

2. Governing equation of motion

In this section, we consider the vibration of an inextensible clamped-free tapered beam as an interesting and important model for engineering structures with inertia and static nonlinearity in the form [26,27]. Fig. 1 illustrates the physical model of the problem.

$$\frac{d^2x}{dt^2} + x + \alpha x^4 \frac{d^2x}{dt^2} + 2\alpha \left(\frac{dx}{dt}\right)^2 x^3 + \beta x^5 = 0,\ x(0) = A,\ \dot{x}(0) = 0. \quad (1)$$

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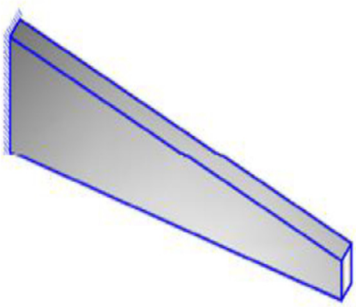

Fig. 1. Geometry of the problem.

Table 1
Comparison of angular frequencies ω_{app} with the exact frequencies ω_{ex} for $\alpha = 1,\ \beta = 1$.

A	ω_2 [26] Error (%)	ω_2 [27] Error (%)	ω Present Error (%)	ω_{ex}
5	2.3701 (1.5821)	2.4397 (1.3080)	2.4613 (2.2050)	2.4082
10	2.3810 (2.5778)	2.4525 (0.3478)	2.4502 (0.2537)	2.4440
15	2.3816 (2.7045)	2.4532 (0.2206)	2.4446 (0.1307)	2.4478
20	2.3817 (2.7401)	2.4533 (0.1838)	2.4495 (0.0286)	2.4488
25	2.3818 (2.7479)	2.4534 (0.1592)	2.4495 (0.0163)	2.4491
30	2.3818 (2.7559)	2.4534 (0.1592)	2.4495 (0.0082)	2.4493
50	2.3818 (2.7638)	2.4534 (0.1592)	2.4495 (0.0000)	2.4495
100	2.3818 (2.7638)	2.4534 (0.1592)	2.4495 (0.0000)	2.4495
200	2.3818 (2.7638)	2.4534 (0.1592)	2.4495 (0.0000)	2.4495
500	2.3818 (2.7638)	2.4534 (0.1592)	2.4495 (0.0000)	2.4495
1000	2.3818 (2.7638)	2.4534 (0.1592)	2.4495 (0.0000)	2.4495
10,000	2.3818 (2.7638)	2.4534 (0.1592)	2.4495 (0.0000)	2.4495

This system describes the unimodal large-amplitude free vibrsations of a slender inextensible cantilever beam carrying an intermediate mass with a rotary inertia. In Eq. (1), the term $(\alpha x^4 (d^2x/dt^2) + 2\alpha (dx/dt)^2 x^3)$ represent inertia type fifth nonlinearity from the inextensibility assumption and the term βx^5 is a static-type fifth nonlinearity due to the potential energy stored in bending.

3. Application of coupled homotopy-variational approach

Consider the nonlinear oscillator Eq. (1), the following homotopy can be constructed:

$$x'' + \omega^2 x + p\left[\alpha x^4 x'' + 2\alpha x'^2 x^3 + \beta x^5 + \left(1 - \omega^2\right)x\right] = 0, \tag{2}$$

when $p = 0$, Eq. (2) becomes the linearized differential equation $x'' + \omega^2 x = 0$, and when $p = 1$, Eq. (2) then becomes the original problem. Suppose that the analytical periodic solution to Eq. (2) can be written as a power series in p:

$$x = x_0 + px_1 + p^2 x_2 + \dots. \tag{3}$$

Now, inserting the above Equation into Eq. (2) and collecting terms with the same powers of p, we obtain:

$$p^0 : \quad x_0'' + \omega^2 x_0 = 0, \tag{4}$$

Table 2
Comparison of angular frequencies ω_{app} with the exact frequencies ω_{ex} for $\alpha = 1,\ \beta = 2$.

A	ω_2 [26] Error (%)	ω_2 [27] Error (%)	ω Present Error (%)	ω_{ex}
5	2.3701 (1.5821)	2.4397 (1.3080)	2.4613 (2.2050)	2.4082
10	2.3810 (2.5778)	2.4525 (0.3478)	2.4502 (0.2537)	2.4440
15	2.3816 (2.7045)	2.4532 (0.2206)	2.4446 (0.1307)	2.4478
20	2.3817 (2.7401)	2.4533 (0.1838)	2.4495 (0.0286)	2.4488
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200	2.3818 (2.7638)	2.4534 (0.1592)	2.4495 (0.0000)	2.4495
500	2.3818 (2.7638)	2.4534 (0.1592)	2.4495 (0.0000)	2.4495
1000	2.3818 (2.7638)	2.4534 (0.1592)	2.4495 (0.0000)	2.4495
10,000	2.3818 (2.7638)	2.4534 (0.1592)	2.4495 (0.0000)	2.4495

$$p^1 : \quad x_1'' + \omega^2 x_1 + \alpha x_0^4 x_0'' + 2\alpha x_0'^2 x_0^3 + \beta x_0^5 + \left(1 - \omega^2\right)x_0 = 0. \tag{5}$$

Solving Eq. (4), we have

$$x_0 = A\ \cos\omega t. \tag{6}$$

The variational approach for x_1 can be obtained as described by [19]:

$$J(x_1) = \int_0^{2\pi/\omega} \left(-\frac{1}{2}x_1'^2 + \frac{1}{2}\omega^2 x_1^2 + \alpha x_0^4 x_0'' x_1 + 2\alpha x_0'^2 x_0^3 x_1 + \beta x_0^5 x_1 + (1 - \omega^2)x_0 x_1 \right) dt. \tag{7}$$

In order to improve the solution accuracy, we define a new trail solution in the form:

$$x_1(t) = B\ (\cos(\omega t) - \cos(3\omega t) - \cos(5\omega t)). \tag{8}$$

By inserting Eq. (8) into Eq. (7), we obtain

$$J(A, B, \omega) = \frac{B\pi\left(-64B\omega^2 - 4A\left(\omega^2 - 1\right) + A^5\left(\alpha\omega^2 + \beta\right)\right)}{4\omega} = 0. \tag{9}$$

Setting

$$\frac{\partial J}{\partial B} = 0, \quad \frac{\partial J}{\partial \omega} = 0. \tag{10}$$

Solving the foregoing equations, we successively achieve the value ω as follow:

$$\omega = \sqrt{\frac{3A^4\beta + 12}{\alpha A^4 - 4}}. \tag{11}$$

Therefore, the solution to the first order approximation can be reformed as follows

$$x = A\cos\left(\sqrt{\frac{3A^4\beta + 12}{\alpha A^4 - 4}}\,t\right). \tag{12}$$

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