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Intuitionistic circular bifuzzy matrices

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In this paper, we define the intuitionistic circular fuzzy matrix and introduce the necessary and sufficient conditions for an intuitionistic fuzzy matrix to be circular. Also, we study some properties of intuitionistic circular fuzzy matrices

1. Introduction

The concept of intuitionistic fuzzy matrices was introduced by Pal et al. [1] as a generalization of the well known ordinary fuzzy matrices introduced by Thomason [2] which take its elements from the unit interval [0,1]. An intuitionistic fuzzy matrix is a pair of fuzzy matrices, namely, a membership and non-membership function which represent positive and negative aspects of the given information (see [3,4]). However, intuitionistic fuzzy matrices have been proposed to represent finite intuitionistic fuzzy relations. This concept is a generalization to that of the ordinary fuzzy relations which also is a generalization to the crisp relations (or Boolean relations).

In this paper, we concentrate oure attention on one of the important kind of intuitionstic fuzzy matrices called intuitionistic circular fuzzy matrices. However, a characterization of intuitionistic circular fuzzy matrices is given and some important properties are established.

The paper is organized in three sections. In Section 2, the definitions and operations on intuitionistic fuzzy matrices are briefly introduced. In Section 3, results concerning of intuitionistic circular fuzzy matrices are proved using the operations and notations in the previous section. In Section 4, we exhibit the adjoint of an intuitionistic circular fuzzy matrix throughout its determinant and show that the adjoint of an intuitionistic circular fuzzy matrix is also circular. However, the operations $\vee$ and $\wedge$ play an important role in our work.

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2. Preliminaries and definitions

We give here some definitions and notations which are applied in the paper. Note that an intuitionistic fuzzy matrix A of order $m \times n$ is defined as follows: $A = \left[a_{ij}\right]$ where $a_{ij} = \left\langle a'_{ij}, a''_{ij}\right\rangle$ and $a'_{ij}, a''_{ij} \in [0, 1]$ maintaining the condition $0 \le a'_{ij} + a''_{ij} \le 1$.

Now, we define some operations on the intuitionistic fuzzy matrices.. For intuitionistic fuzzy matrices $A = \left[a_{ij}\right]_{n\times n}$, $B = \left[b_{ij}\right]_{n\times n}$, and $C = \left[c_{ij}\right]_{n\times m}$ the following operations are defined [3,5–7].

$$
\begin{aligned}
A \wedge B &= \left[a_{ij} \wedge b_{ij}\right] = \left[\langle \min\left(a'_{ij}, b'_{ij}\right), \max\left(a''_{ij}, b''_{ij}\right)\rangle\right],\\
A \vee B &= \left[a_{ij} \vee b_{ij}\right] = \left[\langle \max\left(a'_{ij}, b'_{ij}\right), \min\left(a''_{ij}, b''_{ij}\right)\rangle\right],\\
AC &= \left[\langle \bigvee_{k=1}^{n}\left(a'_{ik} \wedge c'_{kj}\right), \bigwedge_{k=1}^{n}\left(a''_{ik} \vee c''_{kj}\right)\rangle\right],\\
A^k &= \left[a_{ij}^{(k)}\right] = \left[\langle a_{ij}'^{(k)}, a_{ij}''^{(k)}\rangle\right] = A^{k-1}A\\
I_n &= A^0 = \begin{cases} <1, 0> & \text{if } \ i = j,\\ <0, 1> & \text{if } \ i \ne j.\end{cases}\\
A^T &= \left[a_{ji}\right] \ \text{(the transpose of } A),\\
\nabla A &= A \wedge A^T
\end{aligned}
$$

$A \le B$ if and only if $a_{ij} \le b_{ij}$ .That is if and only if $a'_{ij} \le b'_{ij}$ and $a''_{ij} \ge b''_{ij}$ for all i, j.

We may write $\mathbf{0}$ instead of $< 0, 1 >$ and $\mathbf{1}$ instead of $< 1, 0 >$.

Definition 2.1. [1,3,8–11] . For an $n \times n$ intuitionistic fuzzy matrix A we have:

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(a) A is symmetric if and only if $A^T = A$,
(b) A is idempotent if and only if $A^2 = A$,
(c) A is transitive if and only if $A^2 \le A$,
(d) A is circular if and only if $(A^2)^T \le A$,
(e) A is weakly reflexive if and only if $a_{ii} \ge a_{ij}$ for all $1 \le i, j \le n$,
(f) A is reflexive if and only if $a_{ii} = \mathbf{1}$ for all $1 \le i \le n$,
(g) A is similarity if and only if A is symmetric, transitive and reflexive.

It is noted that $(A^T)^2 = (A^2)^T$ for any $n \times n$ matrix. So, the intuitionistic fuzzy matrix A is circular if and only if $A^2 \le A^T$, i.e., $a_{ik} \wedge a_{kj} \le a_{ji}$ for every $1 \le i, j, k \le n$. Moreover, if A is symmetric, then A is transitive if and only if A is circular.

3. Results

Throughout the next two sections we deal only with $n \times n$ intuitionistic fuzzy matrices. In this section, some properties of intuitionistic circular fuzzy matrices are examined by the definitions in the above section. However, we begin with the following proposition.

Proposition 3.1. *Let A be an $n \times n$ intuitionistic fuzzy matrix and let A_1 denotes the $m \times m$ submatrix of A (where $m < n$) such that*

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}.$$

Then A is circular if and only if $A_1^2 \le A_1^T$, $A_2A_3 \le A_1^T$, $A_3A_1 \le A_2^T$, $A_4A_3 \le A_2^T$, $A_1A_2 \le A_3^T$, $A_2A_4 \le A_3^T$, $A_3A_2 \le A_4^T$ and $A_4^2 \le A_4^T$.

Proof. Suppose that A satisfies all the above conditions and consider

$$A^2 = B = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}.$$

Then

$$B_1 = A_1^2 \vee A_2A_3 \le A_1^T \vee A_1^T = A_1^T,$$
$$B_2 = A_1A_2 \vee A_2A_4 \le A_3^T \vee A_3^T = A_3^T,$$
$$B_3 = A_3A_1 \vee A_4A_3 \le A_2^T \vee A_2^T = A_2^T$$

and

$$B_4 = A_3A_2 \vee A_4^2 \le A_4^T \vee A_4^T = A_4^T.$$

Thus, we have $A^2 = B \le A^T$ and A is circular.

Conversely, suppose that A is circular. For $1 \le s \le m$ and $m + 1 \le t \le n$, Let $C = A_1$, $D = A_2$, $E = A_3$ and $F = A_4$. Then $c_{st} = a_{st}$ for every $1 \le s, t \le m$, $d_{st} = a_{s(t+m)}$ for every $1 \le s \le m$ and $1 \le t \le n - m$, $e_{st} = a_{(s+m)t}$ for every $1 \le s \le n - m$ and $1 \le t \le m$ and $f_{st} = a_{(s+m)(t+m)}$ for every $1 \le s \le n - m$ and $1 \le t \le n - m$.

1. To show that $A_1^2 \le A_1^T$ and $A_2A_3 \le A_1^T$, let $G = A_1^2$ and $H = A_2A_3$. Then

$$\begin{aligned} g_{st} &= \langle \bigvee_{k=1}^{m}(c'_{sk} \wedge c'_{kt}), \bigwedge_{k=1}^{m}(c''_{sk} \vee c''_{kt})\rangle \\ &= \langle \bigvee_{k=1}^{m}(a'_{sk} \wedge a'_{kt}), \bigwedge_{k=1}^{m}(a''_{sk} \vee a''_{kt})\rangle \\ &\le \langle \bigvee_{k=1}^{n}(a'_{sk} \wedge a'_{kt}), \bigwedge_{k=1}^{n}(a''_{sk} \vee a''_{kt})\rangle = \langle a'^{(2)}_{st}, a''^{(2)}_{st}\rangle \\ &\le \langle a'_{ts}, a''_{ts}\rangle = a_{ts} = c_{ts}. \end{aligned}$$

Thus, $g_{st} \le c_{ts}$ and therefore, $A_1^2 \le A_1^t$.

Also,

$$\begin{aligned} h_{st} &= \langle \bigvee_{k=1}^{n-m}(d'_{sk} \wedge e'_{kt}), \bigwedge_{k=1}^{n-m}(d''_{sk} \vee e''_{kt})\rangle \\ &= \langle \bigvee_{k=1}^{n-m}(a'_{s(k+m)} \wedge a'_{(k+m)t}), \bigwedge_{k=1}^{n-m}(a''_{s(k+m)} \vee a''_{(k+m)t})\rangle \\ &= \langle \bigvee_{u=m+1}^{n}(a'_{su} \wedge a'_{ut}), \bigwedge_{u=m+1}^{n}(a''_{su} \vee a''_{ut})\rangle \quad (\text{where } u = k + m) \\ &\le \langle \bigvee_{u=1}^{n}(a'_{su} \wedge a'_{ut}), \bigwedge_{u=1}^{n}(a''_{su} \vee a''_{ut})\rangle = \langle a'^{(2)}_{st}, a''^{(2)}_{st}\rangle \\ &\le \langle a'_{ts}, a''_{ts}\rangle = a_{ts} = c_{ts}. \end{aligned}$$

Thus, $h_{st} \le c_{ts}$ and therefore, $A_2A_3 \le A_1^T$.

2. To show that $A_4A_3 \le A_2^T$ and $A_3A_1 \le A_2^T$, let $Q = A_4A_3$ and $L = A_3A_1$. Then

$$\begin{aligned} q_{st} &= \langle \bigvee_{k=1}^{n-m}(f'_{sk} \wedge e'_{kt}), \bigwedge_{k=1}^{n-m}(f''_{sk} \vee e''_{kt})\rangle \\ &= \langle \bigvee_{k=1}^{n-m}\left(a'_{(s+m)(k+m)} \wedge a'_{(k+m)t}\right), \bigwedge_{k=1}^{n-m}\left(a''_{(s+m)(k+m)} \vee a''_{(k+m)t}\right)\rangle \\ &= \langle \bigvee_{u=m+1}^{n}(a'_{(s+m)u} \wedge a'_{ut}), \bigwedge_{u=m+1}^{n}(a''_{(s+m)u} \vee a''_{ut})\rangle \\ &\qquad (\text{where } u = k + m) \\ &\le \langle \bigvee_{u=1}^{n}(a'_{(s+m)u} \wedge a'_{ut}), \bigwedge_{u=1}^{n}(a''_{(s+m)u} \vee a''_{ut})\rangle \\ &= \langle a'^{(2)}_{(s+m)t}, a''^{(2)}_{(s+m)t}\rangle \\ &\le \langle a'_{t(s+m)}, a''_{t(s+m)}\rangle = a_{t(s+m)} = d_{ts}. \end{aligned}$$

Thus, $q_{st} \le d_{ts}$ and therefore, $A_4A_3 \le A_2^T$. Also,

$$\begin{aligned} l_{st} &= \langle \bigvee_{k=1}^{m}(e'_{sk} \wedge c'_{kt}), \bigwedge_{k=1}^{m}(e''_{sk} \vee c''_{kt})\rangle \\ &= \langle \bigvee_{k=1}^{m}(a'_{(s+m)k} \wedge a'_{kt}), \bigwedge_{k=1}^{m}(a''_{(s+m)k} \vee a''_{kt})\rangle \\ &\le \langle \bigvee_{k=1}^{n}(a'_{(s+m)k} \wedge a'_{kt}), \bigwedge_{k=1}^{n}(a''_{(s+m)k} \vee a''_{kt})\rangle \\ &= \langle a'^{(2)}_{(s+m)t}, a''^{(2)}_{(s+m)t}\rangle \le \langle a'_{t(s+m)}, a''_{t(s+m)}\rangle = a_{t(s+m)} = d_{ts}. \end{aligned}$$

i.e., $l_{st} \le d_{ts}$ and therefore, $A_3A_1 \le A_2^T$.

3. To show that $A_1A_2 \le A_3^T$ and $A_2A_4 \le A_3^T$, let $R = A_1A_2$ and $Z = A_2A_4$. Then

$$\begin{aligned} r_{st} &= \langle \bigvee_{k=1}^{m}(c'_{sk} \wedge d'_{kt}), \bigwedge_{k=1}^{m}(c''_{sk} \vee d''_{kt})\rangle \\ &= \langle \bigvee_{k=1}^{m}(a'_{sk} \wedge a'_{k(t+m)}), \bigwedge_{k=1}^{m}(a''_{sk} \vee a''_{k(t+m)})\rangle \\ &\le \langle \bigvee_{k=1}^{n}(a'_{sk} \wedge a'_{k(t+m)}), \bigwedge_{k=1}^{n}(a''_{sk} \vee a''_{k(t+m)})\rangle \\ &= \langle a'^{(2)}_{s(t+m)}, a''^{(2)}_{s(t+m)}\rangle \le \langle a'_{(t+m)s}, a''_{(t+m)s}\rangle = a_{(t+m)s} = e_{ts}. \end{aligned}$$

Therefore, $A_1A_2 \le A_3^T$. Also,

$$\begin{aligned} z_{st} &= \langle \bigvee_{k=1}^{n-m}(d'_{sk} \wedge f'_{kt}), \bigwedge_{k=1}^{n-m}(d''_{sk} \vee f''_{kt})\rangle \\ &= \langle \bigvee_{k=1}^{n-m}(a'_{s(k+m)} \wedge a'_{(k+m)(t+m)}), \bigwedge_{k=1}^{n-m}(a''_{s(k+m)} \vee a''_{(k+m)(t+m)})\rangle \\ &= \langle \bigvee_{u=m+1}^{n}(a'_{su} \wedge a'_{u(t+m)}), \bigwedge_{u=m+1}^{n}(a''_{su} \vee a''_{u(t+m)})\rangle \\ &\le \langle \bigvee_{u=1}^{n}(a'_{su} \wedge a'_{u(t+m)}), \bigwedge_{u=1}^{n}(a''_{su} \vee a''_{u(t+m)})\rangle \\ &= \langle a'^{(2)}_{s(t+m)}, a''^{(2)}_{s(t+m)}\rangle \le \langle a'_{(t+m)s}, a''_{(t+m)s}\rangle = a_{(t+m)s} = e_{ts}. \end{aligned}$$

Hence, $A_2A_4 \le A_3^T$.

4. To show that $A_3A_2 \le A_4^T$ and $A_4^2 \le A_4^T$, let $P = A_3A_2$ and $W = A_4^2$. Then

$$\begin{aligned} p_{st} &= \langle \bigvee_{k=m+1}^{n}(e'_{sk} \wedge d'_{kt}), \bigwedge_{k=m+1}^{n}(e''_{sk} \vee d''_{kt})\rangle \\ &= \langle \bigvee_{k=m+1}^{n}(a'_{(s+m)k} \wedge a'_{k(t+m)}), \bigwedge_{k=m+1}^{n}(a''_{(s+m)k} \vee a''_{k(t+m)})\rangle \\ &\le \langle \bigvee_{k=1}^{n}(a'_{(s+m)k} \wedge a'_{k(t+m)}), \bigwedge_{k=1}^{n}(a''_{(s+m)k} \vee a''_{k(t+m)})\rangle \\ &= \langle a'^{(2)}_{(s+m)(t+m)}, a''^{(2)}_{(s+m)(t+m)}\rangle \end{aligned}$$

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