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# The development of a differential game related to terrorism: Min-Max differential game

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## ABSTRACT

In this work, we study a differential game related to terrorism: Min-Max differential game taking into account the governmental activities such as the education quality, increasing the chances of labor, social justice, religious awareness and security arrangements. A Min-Max differential game between government and terrorist organizations is considered in this study. To obtain the optimal strategy of solving this problem, we study the analytic form of a Min-Max differential game and the governmental activities. Furthermore, a saddle point of a Min-Max differential game is studied.

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## 1. Introduction

Terrorism is the use of violence to cause political, religious and ideological change, or control the wealth of nations. Terrorism problems have become very large and dangerous all over the world. Governments have taken important procedures, such as the education quality, increasing the chances of labor, social justice, religious awareness and security arrangements so as to fight terrorism. Some mathematical subjects are applied to get methods for combating terrorism, particularly 'Operations Research'.

Counter-terrorism measures range from security arrangements and the governmental activities to freezing assets of a terrorist organization or even invading their territories and assassinating them.

However, any of the a fore-mentioned actions has to be thoroughly investigated for consecutive reaction. In this paper, a differential game approach is used for studying the reciprocal strategies of governments on one hand and those terrorist of organizations on the second hand.

The power of organizations is measured by the terrorist attacks, Caulkins et al. [1] proposed that the combating terrorism relies on the community opinion and Caulkins et al. [2] introduced

the comparison between the efficiency of water and fire strategies. The decreasing rate of terrorists is affected by their own actions and the anti-terrorist actions of the government through the education quality, increasing the chances of labor, social justice, religious awareness and security arrangements. The government benefits from the loss of the terrorist resources and their activities but incurs costs for combating terrorism and disutilities cause by terrorist organizations. These organizations try to maximize their power, both by enlarging their size as well as terrorist attacks.

Therefore, this study is an attempt to help governments fight terrorism better effectively. A min-max differential game plays the main role to combat terrorism. Hsie et al. [3,4] introduce the first approach to fuzzy differential game problem: guarding a territory and guarding a movable territory. Youness et al. [5] discuss a parametric-Nash-collative differential game. A study on a fuzzy differential game, a study on large scale continuous differential game, min-max zero-sum fuzzy continuous differential game and min-max zero-sum continuous differential game with fuzzy control are presented in [6–9]. Nova et al. [10] introduce a differential game related to terrorism namely 'Nash and Stackelberg strategies', Roy et al. [11] present a terrorism deterrence in a two country framework: strategic interactions between R&D, defense and preemption. Ahmed et al. [12] introduce a complex adaptive system to study the terrorism phenomena. Megahed [13] presents a Min-Max dif-

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ferential game approach for getting the optimal strategy to fight terrorist organizations.

2. Problem formulation

Consider the following differential game with the state variable,  $x(t)$  which describes the resource of an International Terror Organization (ITO). It may also include weapons, financial capital, network of supporters, etc. and another state variable  $M(t)$  which describe the governmental activities, the education quality, increasing the chances of labor, social justice, religious awareness and security arrangements,  $t \in [0, \infty)$  is the time. Two players are the government with non-negative strategy  $u(t)$  and the other side ITO with non-negative strategy  $v(t)$  as the opponent. The stock of resources of ITO grows according to the growth of a linear function  $g(x)$ , i.e.  $g(x) = rx, r > 0$ , and the governmental activities grows according to a linear function  $A(M) = \mu M$ , where  $\mu > 0$  is the growth rate of the governmental activities. Carrying out attacks make a reduction the growth of the resource stock as it affects negatively the number of terrorists (e.g. due to suicide bombing or terrorists being caught or killed) as well as weapons and financial means, it may even include a reduction of the network of supporters. The reduction of the growth of the resource stock, however does not only depend on the intensity of attacks  $nu(t)$  but it is also influenced by the counter-terror measures  $u(t)$  This influence of the control variables of the two players on the growth of the resource may be denoted as "harvest function"  $h(u, v)$ . As a consequence the dynamics of the resource stock  $x(t)$  can be written as

$$x' = rx(t) - h(u(t), v(t)), \quad x(0) = x_0 > 0 \tag{1}$$

$$M' = \mu M + au - bv \quad M(0) = M_0 > 0 \tag{2}$$

where  $x_0$  denotes the initial stock of terrorist's resources,  $M_0$  is the initial government's activities and  $a, b$  are positive constants. Moreover, we assume that along trajectories the non-negativity constraints

$$x(t) \geq 0, M(t) \geq 0, t \geq 0 \tag{3}$$

Since a higher intensity of counter-terror measure and attacks leads to a reduction of growth, we assume that the partial derivatives are greater than zero  $:h_u(u, v) > 0, h_v(u, v) > 0$ . The counter-terror measures exhibit marginally decreasing efficiency  $h_{uu} < 0$ . Moreover a higher rate of attacks induces disproportional higher losses of resources i.e.  $h_{vv} > 0$ . Finally the instruments reinforce each other, i.e.  $h_{uv} > 0$  which makes economically sense. This positive interaction means that the marginal efficiency of counter-terror activities are increasing with the intensity of terrorist's attacks as active visible terrorists can be more easily controlled than hidden ones. Additionally, we assume that the Inada conditions in the economic literature are fulfilled

$$\lim_{u \rightarrow 0} h_u(u, v) = \infty, \quad \lim_{u \rightarrow \infty} h_u(u, v) = 0 \tag{4}$$

$$\lim_{v \rightarrow 0} h_v(u, v) = 0, \quad \lim_{v \rightarrow \infty} h_v(u, v) = \infty \tag{5}$$

This guarantees that the optimal strategies are nonnegative,  $u(t) \geq 0$ , and  $v(t) \geq 0, t > 0$ .

Player 1 (Government) draw utility from its activities,  $M(t)$  and the loss of the terrorist's resources but disutility from the size of ITO, terrorists activities and their own costs of counter-terror measures. For simplicity all these terms are assumed to be linear. Thus, the objective of the government

$$\max_{u(t)} \left\{ J_1 = \int_0^\infty e^{-\rho_1 t} \times [\omega h(u(t), v(t)) + qM(t) - cx(t) - kv(t) - \alpha u(t)] dt \right\} \tag{6}$$

where  $\omega, c, k, q$  and  $\alpha > 0$ .

The second player (ITO) derives utility from the resource stock  $x(t)$  and the terrorist actions at intensity  $v(t)$  and disutility from Government's activities. This leads to the following maximization problem

$$\max_{v(t)} \left\{ J_2 = \int_0^\infty e^{-\rho_2 t} [\sigma x(t) + \beta v(t) - \omega M(t)] dt \right\} \tag{7}$$

where  $\sigma, \beta, \omega$  and  $\eta > 0$ .

The decreasing rates  $\rho_i, i = 1, 2$  are assumed to be greater than the growth and activity rates  $r, \mu$  respectively i.e.,

$$\rho_i > r, \quad \rho_i > \mu \quad \text{for } i = 1, 2 \tag{8}$$

In this paper, we will calculate min-max equilibrium. The solution procedure relies on Pontryagin's maximum [9].

3. Min-Max equilibrium

A min-max game is called antagonistic game for two persons (two players), In this paper, player 1 is the government and player 2 is the International Terror Organization (ITO). There are two cases for studying of this problem.

3.1. The game of the government view

In this case, the government is going to find the strategic variable  $u(t)$  to maximize its payoff, namely the maximizing player, but the ITO tries to find the strategic variable  $v(t)$  to minimize this payoff, namely the minimizing player. The game takes the following form

$$\left. \begin{aligned} & \min_{v(t)} \max_{u(t)} J_1 \\ & = \int_0^\infty e^{-\rho_1 t} [\omega h(u(t), v(t)) + qM(t) \\ & \quad - cx(t) - kv(t) - \alpha u(t)] dt \\ & x' = rx(t) - h(u(t), v(t)), \quad x(0) = x_0 > 0, x(t) \geq 0 \text{ for all } t \\ & M' = \mu M(t) + au - bv(t), \quad M(0) = M_0 > 0, M(t) \geq 0 \text{ for all } t \end{aligned} \right\} \tag{9}$$

Note: It's denoted that

$$I_1(x(t), u(t), v(t)) = \omega h(u(t), v(t)) + qM(t) - cx(t) - kv(t) - \alpha u(t)$$

and

$$f(x, u, v) = rx(t) - h(u(t), v(t))$$

**Definition 3.1.** The point  $(u^*, v^*)$  is said to be the saddle point of the min-max continuous differential game problem (9) if

$$J_1(u^*, v) \leq J_1(u^*, v^*) \leq J_1(u, v^*) \tag{10}$$

3.2. The necessary conditions of an open saddle point solution

**Theorem 3.1.** Let  $I_1(x(t), u(t), v(t))$  and  $f(x, u, v)$  are continuous differentiable functions. If  $(u^*, v^*)$  is saddle point with the state trajectories  $x^*(t)$  and  $M^*(t)$  for the problem (Government). Then there exists a costate vectors  $\lambda_1(t), P_1(t)$  and the Hamiltonian function  $H_1$  defined by

$$H_1(x(t), u(t), v(t), \lambda_1(t), P_1(t)) = I_1(x(t), u(t), v(t)) + \lambda_1(t)f(x, u, v) + P_1(t)(\mu M + au - bv) \tag{11}$$

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