Journal of the Egyptian Mathematical Society 000 (2017) 1–7

Contents lists available at ScienceDirect

Journal of the Egyptian Mathematical Society

journal homepage: www.elsevier.com/locate/joems

Extended inverse Weibull distribution with reliability application

Hassan M. Okasha [a,b], A.H. El-Baz [c,*], A.M.K. Tarabia [c], Abdulkareem M. Basheer [c,d]

[a] Department of Statistics, Faculty of Science, King Abdul Aziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia
[b] Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City, Cairo 11884, Egypt
[c] Department of Mathematics, Faculty of Science, Damietta University, New Damietta, Egypt
[d] Albayda University, Albayda, Yemen

ARTICLE INFO

Article history:
Received 6 November 2016
Accepted 21 February 2017
Available online xxx

MSC:
62F15
62F30
62F86

Keywords:
Marshall–Olkin extended distributions
Inverse Weibull
Reliability
Moments
Maximum likelihood
Stress-strength

ABSTRACT

The aim of this paper is to introduce an extension of the inverse Weibull distribution which offers a more flexible distribution for modeling lifetime data. We extend the inverse Weibull distribution by Marshall–Olkin method (MOEIW). Some statistical properties of the MOEIW are explored, such as quantiles, moments and reliability. Moreover, the estimation of the MOEIW parameters is discussed by using Maximum Likelihood Estimation method. In addition, the estimation of the stress-strength parameter is discussed. Finally, the proposed extended model is applied on real data and the results are given which illustrate the superior performance of the MOEIW distribution compared to other models.

1. Introduction

The inverse Weibull (IW) distribution has been used to model, many real life applications for example degradation of mechanical components such as pistons, crankshafts of diesel engines, as well as breakdown of insulating fluid [1]. Inverse Weibull distribution with parameters α (scale parameter) and β (shape parameter) with cumulative distribution function and the probability density function of a random variable X are respectively given by

$$F(x) = e^{-\alpha x^{-\beta}}, \quad x \geq 0, \quad \alpha > 0, \quad \beta > 0 \tag{1}$$

$$f(x) = \alpha\beta x^{-(\beta+1)} e^{-\alpha x^{-\beta}}, \quad x \geq 0, \quad \alpha > 0, \quad \beta > 0. \tag{2}$$

Keller et al. [2] obtained the inverse Weibull model by investigating failures of mechanical components subject to degradation. Calabria and Pulcini [3] computed the maximum likelihood and least squares estimates of the parameters of the inverse Weibull distribution. They also obtained the Bayes estimator of the model parameters as well as confidence limits for reliability and tolerance limits, see Calabria and Pulcini [4,5] and Johnson et al. [6] for additional details. Khan et al. [7] presented some important theoretical properties of the inverse Weibull distribution. The generalizations of the inverse Weibull and related distributions with applications are given by Oluyede and Yang [8].

On the other hand, Marshall and Olkin [9] proposed a transformation of the baseline (cdf) by adding a new parameter to obtain a family of distributions

$$G(x;\theta) = \frac{F(x)}{1-\overline{\theta}\,(1-F(x))}$$

$$-\infty < x < \infty, \quad \theta > 0, \quad \overline{\theta} = 1-\theta. \tag{3}$$

Moreover, Marshall–Olkin method is used to obtain new distributions and their properties are studied e.g., Alice and Jose [10] introduced Marshall–Olkin logistic processes, Gui [11] introduced Marshall–Olkin power lognormal distribution and studied its statistical properties of the new distribution. Cordeiro and Lemonte [12] studied some mathematical properties of Marshall–Olkin extended Weibull distribution. Jose and Krishna [13] studied the Marshall–Olkin extended Uniform distribution. Marshall–Olkin Extended Lomax distribution was introduced by Ghitany et al.[14]. Okasha et al. [15] introduced Marshall–Olkin extended

* Corresponding author.
E-mail addresses: hassanokasha@yahoo.com (H.M. Okasha), ali_elbaz@yahoo.com, elbaz@du.edu.eg (A.H. El-Baz), a_tarabia@yahoo.com (A.M.K. Tarabia), abdulkareem_basheer@yahoo.com (A.M. Basheer).

http://dx.doi.org/10.1016/j.joems.2017.02.006
1110-256X/

generalized linear exponential distribution. Marshall–Olkin extended Pareto distribution was introduced by Ghitany [16], Ghitany et al. [17] conducted a detailed study of Marshall–Olkin extended Weibull distribution, that can be obtained as a compound distribution mixing with exponential distribution, and apply it to model censored data.

The rest of the paper is organized as follows: In Section 2, we define our proposed model namely the Marshall–Olkin extended inverse Weibull and its special cases are presented. In Section 3, its reliability analysis is given. In Section 4, its statistical properties are given. In Section 5, the parameters of this model are estimated using Maximum Likelihood Estimation method. The estimation of the stress-strength parameters are discussed in Section 6. Finally, the proposed model is applied on real data and the results are given in Section 7.

2. New model

In this section we will give the Marshall–Olkin extended inverse Weibull (MOEIW) distribution and some of its sub-models.

2.1. MOEIW specification

Let $\Theta = (\alpha, \beta, \theta)$ and by substitution the cumulative function of inverse Weibull given by (1) in Marshall–Olkin given by (3) we get a new distribution denoted as MOEIW (x, Θ) distribution with cdf given by

$$G(x;\Theta) = \frac{e^{-\alpha x^{-\beta}}}{1-\bar{\theta}(1-e^{-\alpha x^{-\beta}})}, \quad x \geq 0, \quad \Theta > 0 \tag{4}$$

which is equivalent to

$$G(x;\Theta) = \frac{e^{-\alpha x^{-\beta}}}{\theta-(\theta-1)e^{-\alpha x^{-\beta}}}, \quad x \geq 0, \quad \Theta > 0. \tag{5}$$

its corresponding probability density function (pdf) is given by

$$g(x;\Theta) = \frac{\alpha\beta\theta x^{-(\beta+1)}e^{-\alpha x^{-\beta}}}{[\theta-(\theta-1)e^{-\alpha x^{-\beta}}]^2}, \quad x \geq 0, \quad \Theta > 0 \tag{6}$$

Fig. 1 gives graphical representation of pdf for different values of α, β and θ.

- Expansion for the density function

For $|z| < 1$ and $\rho > 0$, we have

$$(1-z)^{-\rho} = \sum_{j=0}^{\infty}\frac{\Gamma(\rho+j)}{\Gamma(\rho)j!}z^j \tag{7}$$

where $\Gamma(.)$ is the gamma function.

By using (7) the denominator in (6) can be expressed as

$$(\theta-(\theta-1)e^{-\alpha x^{-\beta}})^{-2} = \frac{1}{\theta^2}\sum_{j=0}^{\infty}(j+1)(1-\frac{1}{\theta})^j e^{-\alpha j x^{-\beta}}$$

Then

$$g(x) = \sum_{j=0}^{\infty}\frac{1}{\theta}(1-\frac{1}{\theta})^j\alpha(j+1)\beta x^{-(\beta+1)}e^{-\alpha(j+1)x^{-\beta}}$$

2.2. MOEIW Sub-models

Some of the sub-models of the MOEIW distribution are listed below:

(i) When θ =1, we have the inverse Weibull (IW) distribution.
(ii) When θ =1 and α=1, we have the Fréchet (F) distribution.
(iii) When θ =1 and β=2, we have the inverse Rayleigh (IR) distribution.
(iv) When θ =1 and β=1, we have the inverse exponential (IE) distribution.

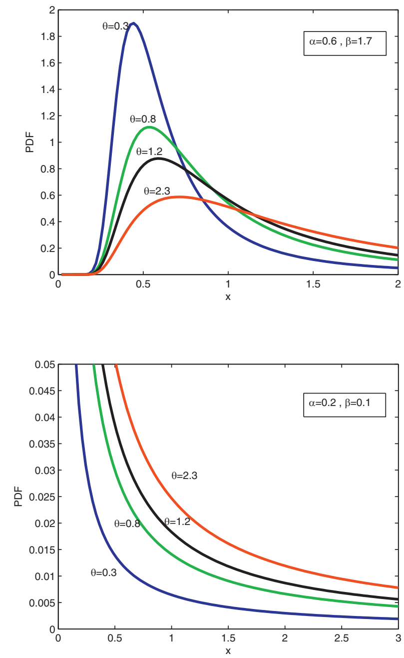

Fig. 1. plots of the PDF of the MOEIW distribution.

3. Reliability analysis

The reliability function (survival function) of MOEIW distribution is given by

$$\bar{G}(x;\Theta) = \frac{\theta(1-e^{-\alpha x^{-\beta}})}{\theta-(\theta-1)e^{-\alpha x^{-\beta}}}, \quad x \geq 0, \quad \Theta > 0. \tag{8}$$

3.1. Hazard rate function

The hazard rate function(failure rate) of a lifetime random variable X with MOEIW distribution is given by

$$h(x;\Theta) = \frac{\alpha\beta x^{-(\beta+1)}e^{-\alpha x^{-\beta}}}{(\theta-(\theta-1)e^{-\alpha x^{-\beta}})(1-e^{-\alpha x^{-\beta}})}, \quad x \geq 0 \tag{9}$$

Fig. 2 gives graphical representations of HRF for different values of α, β and θ.

3.2. Mean residual life

The mean residual life(MRL) function describes the aging process so, it is very important in reliability and survival analysis. The mean residual life(MRL) function of a lifetime random variable X is given by

$$\mu(x) = \frac{1}{\bar{G}(x)}\int_x^{\infty}tg(t)dt - x, \quad x > 0$$

Theorem 3.1. *The MRL function of a lifetime random variable X with MOEIW is given by*

$$\mu(x) = \frac{1}{\bar{G}(x)}\frac{1}{\theta}\times\sum_{j=0}^{\infty}\left(1-\frac{1}{\theta}\right)^j(\alpha(j+1))^{\frac{1}{\beta}}\gamma \times\left(1-\frac{1}{\beta},\alpha(j+1)x^{-\beta}\right) - x, \quad \beta > 1 \tag{10}$$

Proof. From definition of MRL, we get

$$\mu(x) = \frac{1}{\bar{G}(x)}\int_x^{\infty}\frac{1}{\theta}\sum_{j=0}^{\infty}\left(1-\frac{1}{\theta}\right)^j\alpha(j+1)\beta t^{-\beta}e^{-\alpha(j+1)t^{-\beta}}dt - x.$$