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A hybrid numerical method for solving system of high order boundary value problems

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ABSTRACT

Higher order system of boundary value problems arise in several areas of applications. In this paper, we employ the Chebyshev wavelet finite difference method to solve such system of higher order boundary value problems. Numerical experiments are conducted to show the feasibility of the proposed method.

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1. Introduction

Mathematical models of certain problems and phenomena in science and engineering require the solution of system of higher order (≥ 2) boundary value problems (BVPs) involving ordinary differential equations (ODEs). Several new approximate analytical and numerical methods have been developed and applied for various types of problems involving differential equations. Solving system of boundary value problems involving ODEs using new methods has attracted the attention of many researchers. Islam et al. [1] proposed a non polynomial spline approach for the approximate solution of a system of third-order boundary-value problems. Momani [2] employed a modified decomposition method for solving a system of second order obstacle problems. The sinc-collocation method, non-polynomial spline method and variational iteration method were, respectively used in [3–6]. He's homotopy perturbation [7] and B-spline method [8] have also been employed to solve a system of BVPs. Noor et al. [9,10] proposed variational method and the modified variation of parameters method which is a combination of variation of parameters method and Adomian's decomposition method for solving system of second-order and third-order nonlinear boundary value problem. In 2014, Arqub

et al. [11] and Chen et al. [12] applied continuous genetic algorithm and deficient discrete cubic spline methods to obtain the solution to a system of second order boundary value problems. Kazemi Nasab et al. [13,14] solved singular boundary value problems of different types using wavelet analysis method. A composite Chebyshev finite difference method was used for solving singular boundary value problems in [15]. Scalar boundary value problems have been solved by Kazemi Nasab et al. [16,17] using the CWFD method. The operational matrix of fractional integration for shifted Chebyshev polynomials was derived in [18]. Bhrawy et al. [19] proposed a new formula for fractional integral of Chebyshev polynomials. Shifted fractional-order Jacobi orthogonal functions was employed for solving a system of fractional differential equations [20]. Chen et al. [21] applied Legendre wavelets to solve system of nonlinear fractional differential equations. Shifted Jacobi spectral approximations was used for solving fractional differential equations [22]. The question we wish to pursue in this paper is whether CWFD method can be extended for nonlinear higher order systems.

In this paper, we employ CWFD of numerical solution of a system of higher order BVPs of the form:

$$\begin{cases} f_1(x, \mathbf{u}(x), \mathbf{u}^{(1)}(x), \dots, \mathbf{u}^{(\omega-1)}(x), \mathbf{u}^{(\omega)}(x)) = 0, \\ f_2(x, \mathbf{u}(x), \mathbf{u}^{(1)}(x), \dots, \mathbf{u}^{(\omega-1)}(x), \mathbf{u}^{(\omega)}(x)) = 0, \\ \vdots \\ f_r(x, \mathbf{u}(x), \mathbf{u}^{(1)}(x), \dots, \mathbf{u}^{(\omega-1)}(x), \mathbf{u}^{(\omega)}(x)) = 0, \end{cases} \quad (1.1)$$

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E-mail addresses: a.kazeminasab@usm.my, a.kazeminasab@gmail.com (A.K. Nasab).<http://dx.doi.org/10.1016/j.joems.2016.10.002>1110-256X/© 2016 Egyptian Mathematical Society. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license. (<http://creativecommons.org/licenses/by-nc-nd/4.0/>)Please cite this article as: A.K. Nasab et al., A hybrid numerical method for solving system of high order boundary value problems, Journal of the Egyptian Mathematical Society (2016), <http://dx.doi.org/10.1016/j.joems.2016.10.002>

subject to the conditions

$$B_i(u_q(0), \dots, u_q^{(\omega-1)}(0), u_q(1), \dots, u_q^{(\omega-1)}(1)),$$

$$q = 1, 2, \dots, r, i = 1, 2, \dots, r\omega,$$

where $\mathbf{u}^{(\alpha)}(x) = [u_1^{(\alpha)}(x), u_2^{(\alpha)}(x), \dots, u_r^{(\alpha)}(x)]$, $\alpha = 0, 1, 2, \dots, \omega$ and f_j are nonlinear functions of u_j, u'_j, \dots , and $u_j^{(\omega-1)}$, $j = 1, 2, \dots, r$.

2. Wavelets and Chebyshev wavelets

The attractive properties of wavelets in certain situations have persuaded many researchers to consider them as a mathematical tool to solve different kinds of problems arising in mathematics, physics, and engineering. Wavelet analysis can overcome certain shortcomings of Fourier analysis whilst retaining the advantages. The multiresolution analysis aspect of wavelets allows to decomposition of a function or signal into elementary waveforms at different positions and scales to detect important information such as peaks or singularities.

Hence wavelets can be an important tool in the solution of problems involving peaks or singularities. Varying the dilation parameter a and the translation parameter b give rise to the following family of continuous wavelets [23]:

$$\psi_{a,b}(x) = |a|^{-\frac{1}{2}} \psi\left(\frac{x-b}{a}\right), \quad a, b \in \mathbb{R}, a \neq 0. \tag{2.1}$$

Chebyshev wavelets $\psi_{n,m} = \psi(k, n, m, x)$, have five arguments, $n = 1, \dots, 2^{k-1}$, m is degree of Chebyshev polynomials of the first kind, x denotes the time and are defined on $[0, 1)$ as,

$$\psi_{n,m}(x) = \begin{cases} 2^{\frac{k}{2}} p_m T_m(2^k x - 2n + 1), & \frac{n-1}{2^{k-1}} \leq x < \frac{n}{2^{k-1}}, \\ 0, & \text{otherwise,} \end{cases} \tag{2.2}$$

where $p_m, m = 0, 1, \dots, M$ are defined as,

$$p_m = \begin{cases} \frac{1}{\sqrt{\pi}}, & m = 0, \\ \sqrt{\frac{2}{\pi}}, & m \geq 1, \end{cases} \tag{2.3}$$

and T_m are Chebyshev polynomials of the first kind of degree m defined as,

$$T_m(x) = \cos m\beta, \quad \beta = \arccos x, \tag{2.4}$$

which are orthogonal with respect to the weight function $w(x) = 1/\sqrt{1-x^2}$.

A function $f \in L^2[0, 1)$ may be approximated as

$$f(x) \approx \sum_{n=1}^{2^{k-1}} \sum_{m=0}^M c_{n,m} \psi_{n,m}(x) = C^T \Psi(x), \tag{2.5}$$

where C and $\Psi(x)$ are $2^k(M+1) \times 1$ matrices given by

$$C = [c_{1,0}, \dots, c_{1,M}, c_{2,0}, \dots, c_{2,M}, \dots, c_{2^{k-1},1}, \dots, c_{2^{k-1},M}]^T,$$

$$\Psi(x) = [\psi_{1,0}, \dots, \psi_{1,M}, \psi_{2,0}, \dots, \psi_{2,M}, \dots, \psi_{2^{k-1},1}, \dots, \psi_{2^{k-1},M}]^T. \tag{2.6}$$

There are a variety of orthogonal basis functions but some of them have received considerable attention including piecewise constant basis functions, polynomials, and sincosine functions in Fourier series. Chebyshev polynomials are employed to minimize approximation error [24]. Chebyshev wavelets have good characteristics of both Chebyshev polynomials and wavelets. They are very well localized functions so can effectively approximate functions. The multiresolution property of wavelets enables us to adjust the parameters M and k in a proper way to improve the accuracy of solution. Another advantage of Chebyshev wavelets is to convert a given problem to a set of algebraic equations which can be solved easier. With the benefit of sparsity of coefficient matrix, computation performs faster.

3. Chebyshev wavelet finite difference method

A function f can be approximated in terms of a basis of Chebyshev polynomials as follows [25],

$$(P_M f)(x) = \sum_{m=0}^M f_m T_m(x), \tag{3.1}$$

$$f_m = \frac{2}{M} \sum_{k=0}^M f(x_k) T_m(x_k) = \frac{2}{M} \sum_{k=0}^M f(x_k) \cos\left(\frac{mk\pi}{M}\right),$$

where the summation symbol with double primes denotes a sum with both the first and last terms halved. Moreover, the well known Chebyshev-Gauss-Lobatto interpolated points x_m are the extrema of the M th-order Chebyshev polynomial $T_M(x)$ and defined as

$$x_m = \cos\left(\frac{m\pi}{M}\right), \quad m = 0, 1, 2, \dots, M. \tag{3.2}$$

The first three derivatives of the function $f(x)$ at the points $x_m, m = 0, 1, \dots, M$ are given by Elbarbary et al. [26]–[27] as

$$f^{(n)}(x_m) = \sum_{j=0}^M d_{m,j}^{(n)} f(x_j), \quad n = 1, 2, 3 \tag{3.3}$$

where

$$d_{m,j}^{(1)} = \frac{4\gamma_j}{M} \sum_{k=1}^M \sum_{\substack{l=0 \\ (k+l) \text{ odd}}}^{k-1} \frac{k\gamma_k}{c_l} T_k(x_j) T_l(x_m),$$

$$= \frac{4\gamma_j}{M} \sum_{k=1}^M \sum_{\substack{l=0 \\ (k+l) \text{ odd}}}^{k-1} \frac{k\gamma_k}{c_l} \cos\left(\frac{kj\pi}{M}\right) \cos\left(\frac{lm\pi}{M}\right), \tag{3.4}$$

$$d_{m,j}^{(2)} = \frac{2\gamma_j}{M} \sum_{k=2}^M \sum_{\substack{l=0 \\ (k+l) \text{ even}}}^{k-2} \frac{k(k^2-l^2)\gamma_k}{c_l} T_k(x_j) T_l(x_m),$$

$$= \frac{2\gamma_j}{M} \sum_{k=2}^M \sum_{\substack{l=0 \\ (k+l) \text{ even}}}^{k-2} \frac{k(k^2-l^2)\gamma_k}{c_l} \cos\left(\frac{kj\pi}{M}\right) \cos\left(\frac{lm\pi}{M}\right), \tag{3.5}$$

with $\gamma_0 = \gamma_M = \frac{1}{2}, \gamma_j = 1$ for $j = 1, 2, \dots, M-1$, and

$$d_{m,j}^{(3)} = \frac{4\gamma_j}{M} \sum_{k=2}^M \sum_{\substack{l=1 \\ (k+l) \text{ even}}}^{k-2} \sum_{\substack{r=0 \\ (l+r) \text{ odd}}}^{l-1} \frac{kl(k^2-l^2)\gamma_k}{c_l c_r} T_k(x_j) T_r(x_m),$$

$$= \frac{4\gamma_j}{M} \sum_{k=2}^M \sum_{\substack{l=1 \\ (k+l) \text{ even}}}^{k-2} \sum_{\substack{r=0 \\ (l+r) \text{ odd}}}^{l-1} \frac{kl(k^2-l^2)\gamma_k}{c_l c_r} \cos\left(\frac{kj\pi}{M}\right) \cos\left(\frac{rm\pi}{M}\right). \tag{3.6}$$

We are now ready to set up the main idea of this work. Consider $x_{nm}, n = 1, 2, \dots, 2^{k-1}, m = 0, 1, \dots, M$, as the corresponding Chebyshev-Gauss-Lobatto collocation points at the n th subinterval $\left[\frac{n-1}{2^{k-1}}, \frac{n}{2^{k-1}}\right)$ such that,

$$x_{nm} = \frac{1}{2^k} (x_m + 2n - 1). \tag{3.7}$$

On the other hand, a function $f(x)$ can be written in terms of Chebyshev wavelet basis functions as follows

$$(P_M f)(x) \approx \sum_{n=1}^{2^{k-1}} \sum_{m=0}^M c_{nm} \psi_{nm}(x), \tag{3.8}$$

where $c_{nm}, n = 1, 2, \dots, 2^{k-1}, m = 0, 1, \dots, M$, are the expansion coefficients of the function $f(x)$ at the subinterval $\left[\frac{n-1}{2^{k-1}}, \frac{n}{2^{k-1}}\right)$ and

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