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Original Article

Reduced differential transform method for nonlinear integral member of Kadomtsev–Petviashvili hierarchy differential equations

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ABSTRACT

The main objective of this paper is to use the reduced differential to transform method (RDTM) for finding the analytical approximate solutions of two integral members of nonlinear Kadomtsev–Petviashvili (KP) hierarchy equations. Comparing the approximate solutions which obtained by RDTM with the exact solutions to show that the RDTM is quite accurate, reliable and can be applied for many other nonlinear partial differential equations. The RDTM produces a solution with few and easy computation. This method is a simple and efficient method for solving the nonlinear partial differential equations. The analysis shows that our analytical approximate solutions converge very rapidly to the exact solutions.

1. Introduction

Nonlinear partial differential equations (NLPDEs) are mathematical models that are used to describe complex phenomena arising in the world around us. The nonlinear equations appear in many applications of science and engineering such as fluid dynamics, plasma physics, hydrodynamics, solid state physics, optical fibers, acoustics and other disciplines [1]. On the other hand, there are many effective methods for obtaining the anaylatic approximate solutions and exact solutions of NPDEs among of these methods are the inverse scattering method [2], Hirota's bilinear method [3], Backlund transformation [4,5], Painlevé expansion [6] sine–cosine method [7], homogenous balance method [8], homotopy perturbation method [9–12], variation method [13,14], Adomian decomposition method [15,16], tang-function method [17–19], Jacobi elliptic function expansion method [20–23], F-expansion method [24–26] and exp-function method [27–29]. Recently Mahood et al. [30–33] used the optimal homotopy asymptotic to study the MHD slips flow over radiating sheet with heat transfer, the flow heat transfer viscoelastic fluid in an axisymetric channel with a porous wall and for the heat transfer in hollow sphere with the Robin boundary conditions. Also Mahood et al. [34] have discussed the analytical solutions for radiation effects on heat transfer in Blasius flow.

In the present article, we use the reduced differential to transform method (RDTM) which discussed in [35–38], to construct an appropriate solution of some highly nonlinear partial differential equations of mathematical physics. The reduced differential transforms technique is an iterative procedure for obtaining a Taylor series solution of differential equations. This method reduces the size of computational work and easily applicable to many nonlinear physical problems. In this paper, we discuss the analytic approximate solution for two members of the KP hierarchy were formally derived. Two members of the generalized KP hierarchy are given in the following form [39,40]

$$\nu_t = \frac{1}{2}\nu_{xxy} + \frac{1}{2}\partial_x^{-2}[\nu_{yyy}] + 2\nu_x\partial_x^{-1}[\nu_y] + 4\nu\nu_y, \tag{1}$$

and

$$\nu_t = \frac{1}{16}\nu_{xxxxx} + \frac{5}{4}\partial_x^{-1}[\nu\nu_{yy}] + \frac{5}{4}\partial_x^{-1}[\nu_y^2] + \frac{5}{16}\partial_x^{-3}\nu_{yyyy} + \frac{5}{4}\nu_x\partial_x^{-2}\nu_{yy} + \frac{5}{2}\nu\partial_x^{-1}[\nu_{yy}] + \frac{5}{2}\nu_y\partial_x^{-1}[\nu_y] + \frac{15}{2}\nu^2\nu_x + \frac{5}{2}\nu_x\nu_{xx} + \frac{5}{4}\nu\nu_{xxx} + \frac{5}{8}\nu_{xyy} \tag{2}$$

where $\partial_x^{-1} = \int_0^x dx$.

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The two members of the hierarchy (1) and (2) were derived in [39] and [40]. The paper has been organized as follows. Notations and basic definitions are given in Section 2. In Section 3, we apply the RDTM to solve two types of NLPDEs. Conclusions are given in Section 4.

2. Preliminaries and notations

In this section, we give some basic definitions and properties of the reduced differential transform method which are further used in this paper. Consider a function of three variables $u(x, y, t)$ and suppose that it can be represented as a product of three single-variable functions, i.e., $u(x, y, t) = f(x)h(y)g(t)$. Based on the properties the $(2+1)$ of -dimensional differential transform, the function $u(x, y, t)$ can be represented as follows:

$$u(x,y,t) = \left(\sum_{i=0}^{\infty} F(i)x^i\right)\left(\sum_{j=0}^{\infty} H(j)y^j\right)\left(\sum_{l=0}^{\infty} G(l)t^l\right) = \sum_{k=0}^{\infty} U_k(x,y)t^k \tag{3}$$

where $U_k(x, y)$ is called t-dimensional spectrum function of *u(x, y,t)*. The basic definitions of RDTM are introduced as follows [35–38].

Definition 2.1. [35–38] If the function $u(x, y, t)$ is analytic and differentiated continuously with respect to time t and space in the domain of interest, then let

$$U_k(x,y) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,y,t)\right]_{t=0}, \tag{4}$$

where the $t-$ dimensional spectrum function $U_k(x, y)$, is the transformed function. In this paper the lowercase $u(x, y, t)$ represents the original function while the uppercase $U_k(x, y)$ stands for the transform function.

Definition 2.2. [35–38]. The differential inverse transform $U_k(x, y)$ is defined as follows

$$u(x,y,t) = \sum_{k=0}^{\infty} U_k(x,y)\,t^k. \tag{5}$$

Then, combining Eqs. (4) and (5) we have

$$u(x,y,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,y,t)}{\partial t^k}\right]_{t=0} t^k. \tag{6}$$

From the above definitions, it can be found that the concept of the RDTM is derived from the power series expansion. To illustrate the basic concepts of the RDTM, consider the following nonlinear partial differential equation written in an operator form

$$L[u(x,y,t)] + R[u(x,y,t)] + N[u(x,y,t)] = g(x,y,t), \tag{7}$$

with initial condition

$$u(x,y,0) = f(x,y), \tag{8}$$

where $L = \frac{\partial}{\partial t}$, R is a linear operator which has partial derivatives, N is a nonlinear operator and $g(x, y, t)$ is an inhomogeneous term. According to the RDTM, we can construct the following iteration formula:

$$(k+1)U_{k+1}(x,y,t) = G_k(x,y) - R[U_k(x,y)] - N[U_k(x,y)] \tag{9}$$

where $U_k(x, y)$, $R[U_k(x, y)]$,$N[U_k(x, y)]$ and $G_k(x, y)$ are the transformations of the functions $u(x, y, t)$, $R[u(x, y, t)]$,$N[u(x, y, t)]$ and $g(x, y, t)$ respectively. From the initial condition (8), we write

$$U_0(x,y) = f(x,y). \tag{10}$$

Table 1
The fundamental operations of RDTM.

Functional form	Transformed form
$u(x,t)$	$\frac{1}{k!}[\frac{\partial^k}{\partial t^k}u(x,y,t)]_{t=0}$,
$w(x,t) = u(x,t) \pm v(x,t)$	$W_k(x) = U_k(x) \pm V_k(x)$
$w(x,t) = \alpha u(x,t)$	$W_k(x) = \alpha U_k(x,t)$ (α is a constant)
$w(x,t) = x^m t^n$	$W_k(x) = x^m\delta(k-n)$, $\delta(k) = \{^{1k=0}_{0k\neq 0}$
$w(x,t) = x^m t^n u(x,t)$	$W_k(x) = x^m U(k-n)$
$w(x,t) = u(x,t)v(x,t)$	$W_k(x) = \sum_{r=0}^{k} U_r(x)V_{k-r}(x) = \sum_{r=0}^{k} V_r(x)W_{k-r}(x)$
$w(x,t) = \frac{\partial^r}{\partial t^r}u(x,t)$	$W_k(x) = (k+1)\ldots(k+r)W_{k+r}(x) = \frac{(k+r)!}{k!}W_{k+r}(x)$
$w(x,t) = \frac{\partial}{\partial x}u(x,t)$	$W_k(x) = \frac{\partial}{\partial x}U_k(x)$
$w(x,t) = \frac{\partial^2}{\partial x^2}u(x,t)$	$W_k(x) = \frac{\partial^2}{\partial x^2}U_k(x)$.

Substituting (10) into (9) and by straightforward iterative calculation, we get the following $U_k(x, y)$values. Then, the inverse transformation of the set of $U_k(x, y)$, $k = 1, 2, 3, \ldots$ is giving the n- terms approximation solution as follows

$$u_n(x,y,t) = \sum_{k=0}^{n} U_k(x,y)\,t^k. \tag{11}$$

Therefore, the exact solution of the problem is given by

$$u(x,y,t) = \lim_{n\to\infty} u_n(x,y,t). \tag{12}$$

The fundamental mathematical operations performed by RDTM can be readily obtained and are listed in Table 1 [35–38].

3. Numerical results

To demonstrate the effectiveness of the reduced differential transform method (RDTM) algorithm this discussed in the above section. We use this method to construct the analytic approximate solutions for integral member of the generalized KP hierarchy differential equations which have a great attention by many researchers in physics and engineering. The results have been provided by software packages such as Mathematica 9.

Example 3.1. We first consider the KP hierarchy (1) reads:

$$v_t = \frac{1}{2}v_{xxy} + \frac{1}{2}\partial_x^{-2}[v_{yyy}] + 2v_x\partial_x^{-1}[v_y] + 4vv_y. \tag{13}$$

Wazwaz et al have studied the n soliton solution and distinct dispersion of the Kadomtsev–Petviashvili hierarchy in [39, 40]. To get rid of the one and two folds integral operators, we use the transformation

$$v(x,y,t) = u_{xx}(x,y,t), \tag{14}$$

that carries (13) to

$$u_{xxt} = \frac{1}{2}u_{xxxxy} + \frac{1}{2}u_{yyy} + 2u_{xxx}u_{xy} + 4u_{xx}u_{xxy}, \tag{15}$$

with the initial condition

$$u(x,y,0) = Ln(1+e^{kx+ry}), \tag{16}$$

Or

$$v(x,y,0) = \frac{k^2e^{kx+ry}}{(1+e^{kx+ry})^2}, \tag{17}$$

where k and r are an arbitrary constant.

Applying the reduced differential transform to the Eq. (13), we obtain the following iteration relation,

$$(k+1)V_{k+1}(x,y) = \frac{1}{2}\frac{\partial^3 V_k(x,y)}{\partial x^2\partial y} + \frac{1}{2}\partial_x^{-2}\left[\frac{\partial^3 V_k(x,y)}{\partial y^3}\right] + 2B_k(x,y) + 4C_k(x,y), \tag{18}$$