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Solitons and other solutions to a new coupled nonlinear Schrodinger type equation

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ABSTRACT

In this paper, the first integral method combined with Liu's theorem is applied to integrate a new coupled nonlinear Schrodinger type equation. Using this combination, more new exact traveling wave solutions are obtained for the considered equation using ideas from the theory of commutative algebra. In addition, more solutions are also obtained via the application of semi-inverse variational principle due to Ji-Huan He. The used approaches with the help of symbolic computations via Mathematica 9, may provide a straightforward effective and powerful mathematical tools for solving nonlinear partial differential equations in mathematical physics.

1. Introduction

In recent years, the investigation of exact solutions to nonlinear partial differential equations (NPDEs.) has played an important role in nonlinear phenomena. Nonlinear phenomena appear in a wide variety of scientific applications such as plasma physics, solid state physics and fluid dynamics .In order to better understanding these nonlinear phenomena, many mathematicians as well as physicists have been made big efforts to seek more exact solutions to NPDEs. Therefore, several powerful methods have been proposed to obtain exact solutions of nonlinear equations, such as inverse scattering method [1], Backlund transformation method [2], Hirota direct method [3,4], tanh-sech method [5–7], extended tanth method [8–10], $(G'/G, 1/G)$ - expansion method [11], modified simplest equation method [12,13], homogeneous balance method [14,15], Jacobi elliptic function expansion method [16], F- expansion method [17], the transformed rational function method [18] and others.

The first integral (FI) method was first proposed by Feng in [19] in solving Burgers-KdV equation which is based on the ring theory of commutative algebra. Recently, this useful method has been widely used by many authors such as [20–25] and by the references therein.

The variational approaches such as Ji-Huan He semi-inverse variational (SIV) method [26] is a powerful mathematical tool for searching the variational principles of nonlinear physical systems from the field equations without using Lagrange multipliers.

Yong et al. [27], have studied the following new coupled nonlinear Schrodinger type (CNLST) equation

$$\begin{cases} u_{xt} = u_{xx} + \dfrac{2}{1-\beta^2}|u|^2\ u + u\,(v-w), \\ v_t = -\dfrac{(|u|^2)_t}{1+\beta} + (1+\beta)\,v_x, \\ w_t = \dfrac{(|u|^2)_t}{1-\beta} + (1-\beta)\,w_x, \end{cases} \tag{1}$$

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by using the truncated singular expansions and direct quadrature method to obtain exact solutions of this equation.

The new coupled nonlinear Schrodinger type (CNLST) Eq. (1) was proposed in (2009) by Ma and Geng via spectral problem and its auxiliary one [28]. In this present paper, we aim to extend the previous works made in [27], to extract more exact solutions of the new coupled nonlinear Schrodinger type (CNLST) Eq. (1) via two distinct algorithms, namely the first integral(FI) method combined with Liu's theorem and Ji-Huan He's semi-inverse variational (SIV) method.

The layout of this paper is as follows: in Section 2 we present basic algorithm of the first integral (FI)method .In Section 3, application to the new coupled nonlinear Schrodinger type (CNLST) Eq. (1) is considered. Also, the algorithm of semi-inverse variational (SIV) method combined with its application to the considered equation are presented in Sections 4 and 5. The graphics of the obtained solutions accompanied with their explanations have been added in Section 6. Section 7 is devoted to some conclusions.

2. Algorithm of the FI method

Consider a general nonlinear partial differential equation (nonlinear PDE) in the form

$$F(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}, u_{xxx}, \dots) = 0, \tag{2}$$

where $u = u(x,t)$ is the solution of this nonlinear PDE (2).

We use the traveling wave transformation

$$u(x,t) = u(\xi), \tag{3}$$

where $\xi = x - \lambda t + \xi_0$, and ξ_0 is an arbitrary constant. This enables us to use the following changes:

$$\frac{\partial}{\partial t}(\bullet) = -\lambda\frac{\partial}{\partial \xi}(\bullet), \quad \frac{\partial}{\partial x}(\bullet) = \frac{\partial}{\partial \xi}(\bullet),$$
$$\frac{\partial^2}{\partial x^2}(\bullet) = \frac{\partial^2}{\partial \xi^2}(\bullet), \cdots . \tag{4}$$

Using Eq. (4), the nonlinear PDE (2) is transformed to the nonlinear ordinary differential equation (nonlinear ODE)

$$G(u(\xi), \partial u(\xi)/\partial\xi, \partial^2 u(\xi)/\partial\xi^2, \dots) = 0. \tag{5}$$

Next, we introduce new independent variables

$$X(\xi) = u(\xi), \quad Y(\xi) = \partial u(\xi)/\partial\xi, \tag{6}$$

which lead to a system of nonlinear ODEs.:

$$\partial X(\xi)/\partial\xi = Y(\xi) \tag{7a}$$

$$\partial Y(\xi)/\partial\xi = F(X(\xi), Y(\xi)) \tag{7b}$$

According to the qualitative theory of ordinary differential equations [29], if we can find two first integrals to system (7) under the same conditions, then analytic solutions to Eqs. (7a) and (7b) can be solved directly. However, in general, it is difficult to realize this even for one first integral, because for a given plane autonomous system, there is no systematic theory that can tell us how to find its first integrals, nor is there a logical way for telling us what these first integrals are.

We will apply the Division theorem to obtain one first integral to Eqs. (7a) and (7b) which reduces Eq. (5) to a first order integrable ODE. An exact solution to Eq. (2) is then obtained by solving this equation.

For convenience, first let us recall the Division theorem.

Theorem 1 (Divison theorem)**.** *Suppose that $P(w,z)$ and $Q(w,z)$ are polynomials in $C(w,z)$ and $P(w,z)$ is irreducible in $C(w,z)$. If $Q(w,z)$ vanishes at all zero points of $P(w,z)$, then there exists a polynomial $G(w,z)$ in $C(w,z)$ such that*

$$Q(w,z) = P(w,z)\,G(w,z). \tag{8}$$

The Division theorem follows immediately from the Hilbert–Nullstellensatz Theorem [30], namely,

Theorem 2 (Hilbert – Nullstellensatz theorem)**.** *Let k be a field and L an algebraic closure of k.*

(1) Every ideal γ of $k[X_1, \dots, X_n]$ not containing 1 admits at least one zero in L^n
(2) Let $x = (x_1, \dots, x_n)$, $y = (y_1, \dots, y_n)$ be two elements of L^n; for the set of polynomials of $k[X_1, \dots, X_n]$ zero at x to be identical with the set of polynomials of $k[X_1, \dots, X_n]$ zero at y, it is necessary and sufficient that there exists a $k-$automorphism s of L such that $y_i = s(x_i)$ for $1 \le i \le n$.
(3) For an ideal α of $k[X_1, \dots, X_n]$ to be maximal, it is necessary and sufficient that there exists an x in L^n such that α is the set of polynomials of $k[X1_1, \dots, X_n]$ zero at x.
(4) For a polynomial Q of $k[X_1, \dots, X_n]$ to be zero on the set of zeros in L^n of an ideal γ of $k[X_1, \dots, X_n]$, it is necessary and sufficient that there exists an integer $m \succ 0$ such that $Q^m \in \gamma$.

Theorem 3 (Liu's theorem [31])**.** *If Eq. (2) has a kink-type solution*

$$u(\xi) = Q_\ell(\tanh[A(\xi + \xi_0)]), \tag{9}$$

then, it has certain kink-bell - type solution

$$u(\xi) = Q_\ell(\tanh[2A(\xi + \xi_0)] \pm i\,\mathrm{sec}\,h[2A(\xi + \xi_0)]), \tag{10}$$

where Q_ℓ is a polynomial of degree k, i is the imaginary number, namely, $i = \sqrt{-1}$.

3. Application

The authors in [27] have taken the traveling wave transformation [32]

$$u = \phi(\xi)\ e^{i(kx-\omega t)}, \quad v = V(\xi), \quad w = W(\xi),$$
$$\xi = x - \lambda t + \xi_0, \tag{11}$$

where ξ_0 is an arbitrary constant. Conducting the analysis made in [27] on the new coupled nonlinear Schrodinger type (CNLST) Eq. (1), thus, the following results have been obtained as

$$V(\xi) = -\frac{\lambda\phi^2}{(\beta+1)(\beta+\lambda+1)}, \quad W(\xi) = \frac{\lambda\phi^2}{(\beta-1)(\beta-\lambda-1)} \tag{12}$$

and the reduced nonlinear ODE

$$\phi'' = -k^2\phi - \frac{2}{(\lambda+1+\beta)\ (\lambda+1-\beta)}\phi^3 \tag{13}$$

where $' := d/d\xi$.

Therefore, we are concerned to solve the Lienard Eq. (13).

By introducing new independent variables $X = \phi(\xi)$ and $Y = \phi'(\xi)$ and using (6), we get a system of nonlinear ODEs

$$X'(\xi) = Y(\xi) \tag{14a}$$

$$Y'(\xi) = (-k^2)X(\xi) - \left(\frac{2}{(\lambda+1+\beta)\ (\lambda+1-\beta)}\right)X^3(\xi). \tag{14b}$$

According to the first integral method, we suppose that $X(\xi)$ and $Y(\xi)$ are the nontrivial solutions of (14a) and (14b), and

$$Q(X,Y) = \sum_{i=0}^{m} a_i(X)\,Y^i, \tag{15}$$

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