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Original Article

Melting heat and mass transfer in stagnation point micropolar fluid flow of temperature dependent fluid viscosity and thermal conductivity at constant vortex viscosity

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ABSTRACT

Steady mixed convection micropolar fluid flow towards stagnation point formed on horizontal linearly stretchable melting surface is studied. The vortex viscosity of micropolar fluid along a melting surface is proposed as a constant function of temperature while dynamic viscosity and thermal conductivity are temperature dependent due to the influence of internal heat source on the fluid. Similarity transformations were used to convert the governing equation into non-linear ODE and solved numerically. A parametric study is conducted. An analysis of the results obtained shows that the flow-field is influenced appreciably by heat source, melting, velocity ratio, variable viscosity and thermal conductivity.

1. Introduction

Within the last few decades, many researchers have reported the behavior of fluid flow within a thin layer formed on a stretchable surface in the presence of pressure gradient. The study of stagnation point flow was pioneered by Hiemenz [1]. Stagnation point flow appears in virtually all fields of science and engineering. Shateyi and Makinde [2] stated that a flow can be stagnated by a solid wall or a stagnation point in the interior of the fluid domain. For more related studies on stagnation point flow, prediction of skin-friction and heat/mass transfer near stagnation regions see Refs. [3–6]. Realistically, during the industrial production of polymer fluids, colloidal solutions and fluid containing small additives; there is often a point where the local velocity of the fluid possesses symmetric stress tensor and micro-rotation of particles is zero. Some fluids possess microstructure and belong to class of fluid with nonsymmetric stress tensor. This kind of fluid consists of rigid, randomly oriented particles suspended in a viscous medium; see Lukaszewicz [7]. Micropolar fluid supports couple stress and distributed body torque which cannot be accurately study by using classical Navier–Stokes equation or the viscoelastic flow models. Eringen in [8,9] started an analysis on the theory of micropolar fluids which provided a mathematical model for its non-Newtonian behavior. Recently, Sandeep et al. [10] adopted the idea and reported the effect of radiation on a stagnation point flow of micropolar fluid over a nonlinearly stretching surface. It is a well-known fact in the field of fluid dynamics that static pressure is highest when the velocity is zero and hence static pressure is at its maximum value at stagnation points. In most cases, engineers in the industry tend to introduce internal heat generation to reduce drag and enhance easy flow of fluid around stagnation point where the velocity is zero. Internal energy generation can be explained as a scientific method of generating heat energy within a body by a chemical, electrical or nuclear process. Natural convection induced by internal heat generation is a common phenomenon in nature. Crepeau and Clarksean [11] carried out a similarity solution for a fluid with an exponentially decaying heat generation term. However, micropolar fluid flow towards a stagnation point on a melting surface is significant. In the presence of space heat source,

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dynamic viscosity and thermal conductivity may certainly vary with temperature whereas the vortex viscosity may never be influenced or influenced infinitesimally.

From the knowledge of kinetic theory of matter, every solid melts if expose to a high temperature. In an earlier study, the effect of melting on heat transfer was studied by Tien and Yen [12]. In recent years, many researchers have investigated and reported the effect of melting parameters. For more details, see Refs. [13–16]. In all of the above mentioned studies, fluid viscosity and thermal conductivity was assumed to be constant function of temperature within the boundary layer. However, it is known that the physical properties of the fluid may change significantly when expose to internal generated temperature. For lubricating fluids, heat generated by the internal friction and the corresponding rise in temperature affect the viscosity of the fluid and so the fluid viscosity can no longer be assumed constant. In a case of melting as reported by many researchers; it is important to notice that temperature of fluid layers at free stream may also have significant effect on the intermolecular forces of the micropolar fluid. The increase of temperature may also leads to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so the heat transfer rate at the wall may also be affected greatly. According to Refs. [17,18] and Meyers et al. [19], it is a well-known fact that properties of fluid which are most sensitive to an increase in temperature are viscosity and thermal conductivity. Considering this concept, effects of temperature-dependent viscosity and variable thermal conductivity on unsteady MHD flow past an impulsively started vertical surface and MHD non-Darcy mixed convective diffusion of species over a stretching sheet was considered in [20,21]. Motivated by all the works mentioned above, it is of interest to extend the work of [4,22,23] by including such effects on the flow and also consider the diffusion of species (mass) in micropolar fluid flow over a melting surface. This is to further examine a case in which the vortex viscosity of micropolar fluid is negligibly influenced due to the nature of wall temperature in the case of melting heat transfer.

2. Problem formulations

Steady laminar incompressible flow, heat and mass transfer of a micropolar fluid towards a horizontal linearly stretching melting surface is considered. It is assumed that the temperature of the melting surface is T_m while the temperature in the free-stream is T_∞ such that $T_m < T_\infty$. Consequently, the species/mass of the micropolar fluid at the melting wall C_m and at the free stream C_∞ satisfies $C_m < C_\infty$. The temperature and concentration of the solid far from the interface is $T_o(<T_m)$ and $C_o(<C_m)$ respectively. The x-axis is along the melting surface while y-axis is normal to it. It is assumed that the stretching of fluid layer at the free stream (i. e. region of inviscid) is $u_e \to ax$ and stretching velocity of the melting surface $u_w = cx$ where both a and c are known as stretching index with unit s^{-1}. Positive values of a and c corresponds to stretching of the surface while x measures the distance along the surface of the plate. Two equal and opposite forces are introduced along x-axis so that the horizontal melting wall is stretched keeping the origin fixed. This external force induces the fluid to flow in x direction. According to Sir Isaac Newton, the differential form of viscous forces

$$\tau^* = \frac{F}{A} = \mu\frac{\partial u}{\partial y}.$$

Where the local shear velocity is $\frac{\partial u}{\partial y}$ and μ is known as constant of proportionality. Since $\tau^* = \mu\frac{\partial u}{\partial y}$, this formulae assumes that the fluid satisfies all the conditions of Couette flow along a parallel lines and y axis perpendicular to the flow, points in the direction of maximum shear velocity. Upon using the scaling analysis (order of magnitude) according to Ludwig Prandtl, this often leads to the simplification of the remaining viscous term of momentum equation as

$$\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{1}$$

when investigating a case in which viscosity of the fluid flow vary with temperature due to the correlation between the two concepts (i.e. variation of viscosity due to pressure gradient as in the case of Couette flow and variation of viscosity due to temperature). In a case of micropolar fluid where the addition of dynamic viscosity and vortex viscosity plays important role in the modeling of deviatoric stress tensor, it may not be realistic to impose the same condition as in Eq. (1) on the fluid flow over a melting surface. Likewise, it may not be valid to neglect the influence of temperature on the dynamic viscosity of micropolar fluid. It is very important to note that base on this fact, the vortex viscosity might not be influenced the same way with dynamic viscosity. A vortex is a region in a fluid medium in which the flow is mostly rotating around an axis line, the vortical flow that occurs either on a straight axis or a curved axis Loper [24] and Ref. [18]. Examples include whirlpools in the smoke rings, dust devil, wake of a boat, paddle or aeroplane. It is important to also note that both vortex and rotation of micro-elements may be restrained near the wall which possesses low heat energy. In view of this, vortex viscosity is assumed to be constant function of temperature. There are several models for shear viscosity e.g. exponential model, Arrhenius model, Williams Landel–Ferry model, Masuko–Magill model and Batchelor model. All these models were developed for either liquid or gases in which vortex viscosity is zero or totally neglected or out of consideration. Under the usual boundary-layer approximations, the basic equations taking into account the presence of internal heat generation in the energy equation for a micropolar fluid can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right) + \frac{\tau}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{\tau}{\rho}\frac{\partial N}{\partial y} - \left(\frac{\mu+\tau}{\rho\delta}\right)(u_e - u), \tag{3}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma^*}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{\tau}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right), \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(\kappa(T)\frac{\partial T}{\partial y}\right) + \frac{\kappa(T)a}{C_p\mu(T)}\left[A^*(T_\infty - T_m)e^{-y\sqrt{\frac{a}{\vartheta}}} + B^*(T - T_m)\right], \tag{5}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2}. \tag{6}$$

The appropriate boundary conditions on velocity, micro-rotation and temperature are

$$u_w = cx, \quad \kappa(T)\frac{\partial T}{\partial y} = \rho[\lambda^* + c_s(T_m - T_o)]v(x,0),$$
$$N = -m_o\frac{\partial u}{\partial y}, \quad T = T_m, \quad C = C_m \quad at \quad y = 0, \tag{7}$$

$$u_e \to ax, \quad N \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad as \quad y \to \infty. \tag{8}$$

The formulation of $\kappa(T)\frac{\partial T}{\partial y} = \rho[\lambda^* + c_s(T_m - T_o)]v(x,0)$ in Eq. (7) states that the heat conducted to the melting surface