Journal of the Egyptian Mathematical Society (2016) **000**, 1–9

Egyptian Mathematical Society
Journal of the Egyptian Mathematical Society
www.etms-eg.org
www.elsevier.com/locate/joems

Original Article

A numerical scheme for the generalized Burgers–Huxley equation

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Received 23 July 2014; revised 11 August 2015; accepted 19 November 2015
Available online xxx

Keywords
Modified cubic B-spline;
gBH equation;
SSP-RK43 scheme;
Thomas algorithm

Abstract In this article, a numerical solution of generalized Burgers–Huxley (gBH) equation is approximated by using a new scheme: *modified cubic B-spline differential quadrature method* (MCB-DQM). The scheme is based on *differential quadrature method* in which the weighting coefficients are obtained by using modified cubic B-splines as a set of basis functions. This scheme reduces the equation into a system of first-order *ordinary differential equation* (ODE) which is solved by adopting SSP-RK43 scheme. Further, it is shown that the proposed scheme is stable. The efficiency of the proposed method is illustrated by four numerical experiments, which confirm that obtained results are in good agreement with earlier studies. This scheme is an easy, economical and efficient technique for finding numerical solutions for various kinds of (non)linear physical models as compared to the earlier schemes.

1. Introduction

The (generalized) Burgers–Huxley equation describes a wide class of physical nonlinear phenomena, for instance, a prototype model for describing the interaction between reaction mechanisms, convection effects and diffusion transports [42]. It have finds its applications in many fields such as biology, metallurgy, chemistry, metallurgy, combustion, mathematics and engineering [20,42]. In this article, we concerned with the numerical solution of one dimensional gBH equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \alpha u^\delta \frac{\partial u}{\partial x} + \beta u f_{\delta,\gamma}(u), x \in \Omega, t \geq 0, \quad (1.1)$$

with the initial condition: $u(x, 0) = g(x), x \in \Omega$ and boundary conditions: $u(x, t) = \psi_x(t), x \in \partial\Omega, t > 0,$ where $\Omega = (a, b)$ and $f_{\delta,\gamma}(u) = (1 - u^\delta)(u^\delta - \gamma)$ is a nonlinear reaction term.

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Peer review under responsibility of Egyptian Mathematical Society.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.joems.2015.11.003

The coefficient $\beta \geq 0$ and α are reaction and advection coefficient, respectively, $0 < \gamma < 1$ and $\delta > 0$. *Huxley equation* (Eq. (1.1) with $\alpha = 0, \delta = 1$) was proposed to explain the ionic mechanisms underlying the initiation and propagation of action potentials in the squid giant axon. Eq. (1.1) with $\alpha = 0$ correspond to the well known Fitz-hugh–Nagoma equation [14], and for $\beta = 0$ Eq. (1.1) becomes a modified Burgers equation [7].

In the recent years, many numerical techniques to approximate nonlinear time dependent partial differential equations have been designed, several technics have been designed to such type of differential equations, see [2,9,23,32,38–40,44,48] and references therein. gBH equation have been studied theoretically/numerically by adopting various techniques. The solitary wave solutions of gBH equation are obtained by Wang et al. [47] using nonlinear transformation. The kink wave solution of gBH Eq. (1.1) as presented in [47] is given by

$$u(x,t) = \left[\frac{\gamma}{2}\{1 + \tanh(k(x - ct))\}\right]^{\frac{1}{\delta}}, \tag{1.2}$$

where the parameters c and k:

$$c = \frac{\alpha\gamma}{\delta + 1} - \frac{(1 + \delta - \gamma)\left[-\alpha + \sqrt{\alpha^2 + 4\beta(\delta + 1)}\right]}{2(\delta + 1)} \quad \text{and}$$
$$k = \frac{\gamma\delta\left[-\alpha + \sqrt{\alpha^2 + 4\beta(\delta + 1)}\right]}{4(\delta + 1)}$$

are the velocity and the wave number respectively.

The solitary wave solutions of the BH equation are obtained by Wazwaz [48] using the tanh-coth method, by Ismail et al. [22] and Hashim et al. [18,19] using Adomian decomposition method (ADM), by Bataineh et al. [4] and Molabahrami and Khami [35] using the homotopy analysis method. The travelling wave solutions for gBH equation were derived by Efimova and Kudryashov [11] using Hope-Cole transformation, by Batiha et al. [3] using variational iteration method (VIM) and by Gao and Zhao [15] using He's Exp-function method. The travelling wave analysis for BH equation have been reported by Griffiths and Schiesser [16]. A class of travelling solitary wave solutions for the gBH equation are obtained by Deng [10].

A large number of techniques have been developed for the numerical simulation of nonlinear BH equation, for instance, spectral collocation method [9,25,26], new domain decomposition algorithm based on Chebyshev polynomials (DDAC) [28], high order finite difference schemes [41], Chebyshev spectral collocation with the domain decomposition [27], a fourth-order finite difference scheme (FDS4) [8], spectral method (SM) [21,23], nodal Galerkin (Gauss Chebyshev Galerkin (GCG), El-Gendi Chebyshev Galerkin (ECG) and El-Gendi Legendre Galerkin (ELG)) methods [29], differential quadrature method (DQM) [32], optimal Homotopy asymptotic method (OHAM) [36], Homotopy analysis method [5], a monotone finite difference scheme [13], B-spline collocation method [33]. Dehghan et al. [12] derived new methods based on the interpolation scaling functions and the mixed collocation difference schemes for the solution of gBH equation. Gupta and Kadalbajoo [17] constructed a monotone finite difference operator for the singularly perturbed Burgers–Huxley equation, it is a natural development of monotone ϵ-convergent schemes for linear boundary value problems with exponential boundary layer. Zhou and Cheng [49] developed a linearly semi-implicit compact scheme for the BH equation with the help of time-splitting method. Recently, Mohanty et al. [34] developed a new two-level implicit operator compact method with accuracy of order two in time and four in space for the numerical simulation of time dependent BH equation.

DQM [6] has been widely used for numerical simulation of a number of (non)linear physical problems. In DQM the weighting coefficients are evaluated using various test functions: *spline functions*, *Lagrange interpolation polynomials, cubic B-splines, modified cubic B-splines and sinc function*, see [1,30,31,37,43,45] and references therein. This article present a numerical solution of gBH Eq. (1.1) approximated by using a new scheme: MCB-DQM [1]. The scheme is based on DQM where modified cubic B-splines are used as basis functions. On implementing DQM, the gBH equation is reduced into a system of first-order ODEs. Keeing stability criteria in mind, SSP-RK43 [46] scheme is used to solve the resulting system of ODEs. The proposed results are computed without using any transformation and linearization process. The efficiency of the proposed method is confirmed by four test problems.

This paper is organized as follows. In Section 2, the description of the modified cubic B-spline differential quadrature method is given. In Section 2.2, procedure for implementation of method is described. Four test problems are illustrated to establish the applicability and accuracy of the proposed method in Section 3. Section 4 concludes the article.

2. Description of modified cubic B-spline DQM method

The modified cubic B-spline differential quadrature method is the differential quadrature method (DQM is approximation to the derivatives of a function using the weighted sum of the functional values at certain discrete points) in which the weighting coefficients are obtained by using modified cubic B-spline functions as a set of basis functions. Since the weighting coefficients are dependent on the spatial grid spacing only, one can assume N grid points on the real axis distributed uniformly, that is, $a = x_1 < x_2, \ldots, x_{N-1} < x_N = b$ with $x_{i+1} - x_i = h$. The solution $u(x, t)$ at any time on the knot x_i is $u(x_i, t)$ for $i = 1, \ldots, N$. The approximate value of first and second order spatial derivatives are given by

$$\left.\frac{\partial u}{\partial x}\right|_{x=x_i} = \sum_{j=1}^{N} a_{ij} u(x_j, t), \quad \left.\frac{\partial^2 u}{\partial x^2}\right|_{x=x_i} = \sum_{j=1}^{N} b_{ij} u(x_j, t), \quad i = 1, \ldots, N \tag{2.1}$$

where a_{ij} and b_{ij} are weighting coefficients of the first and second order derivatives with respect to x, respectively [6].

The cubic B-spline basis functions at the knots are defined as follows

$$\varphi_j(x) = \frac{1}{h^3}\begin{cases} (x - x_{j-2})^3 & x \in [x_{j-2}, x_{j-1}) \\ (x - x_{j-2})^3 - 4(x - x_{j-1})^3 & x \in [x_{j-1}, x_j) \\ (x_{j+2} - x)^3 - 4(x_{j+1} - x)^3 & x \in [x_j, x_{j+1}) \\ (x_{j+2} - x)^3 & x \in [x_{j+1}, x_{j+2}) \\ 0 & \text{otherwise,} \end{cases} \tag{2.2}$$

where $\{\varphi_0, \varphi_1, \ldots, \varphi_N, \varphi_{N+1}\}$ forms a basis over the region $[a, b]$.