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Original Article

Lie point symmetries for a magneto couple stress fluid in a porous channel with expanding or contracting walls and slip boundary condition

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Abstract In this paper, an incompressible couple stress fluid flow with magnetic field in a porous channel with expanding or contracting walls and slip boundary condition is considered. Lie group analysis and group invariant solutions are obtained, the governing equations are reduced to nonlinear ordinary differential equations. The resulting equations are solved analytically, also this equations are solved by using Adomian method. The graphs for the axial and the normal velocity components and the pressure distribution for different values of the physical and geometric parameters are plotted and discussed. Finally, the comparison between the analytic and Adomian methods is discussed. It is found that for no-slip case $\phi = 0$ the fluid adheres to the walls and axial velocity is maximum at the center of the channel, also by increasing the slip parameter the velocity at the channel walls increases. However, it decreases at center of the channel by increasing slip parameter.

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1. Introduction

Studying of a couple stress fluid is an important to understanding some physical problems, because it have the mechanism to characterize rheologically complex fluids such as liquid crystals, colloidal fluids, animal and human blood and lubrication. The micro-continuum theory of couple stress fluid proposed by Stokes [1], some theoretical studies [2–4] considered the blood couple stress fluid flow as a non-Newtonian fluid flow for its properties. Sometimes the couple-stress fluid considered as a special case of a non-Newtonian fluid which is purposed to take the effect of the particle size into account. Moreover, the couple stress fluid model is one of the numerous models that proposed to show response characteristics of non-Newtonian fluids. The constitutive equations for couple stress fluid models is very complex and involving number of parameters, also

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the out coming couple stress equations lead to boundary value problems with higher order than the Navier–Stokes. Shehawey and Mekheimer [5] proposed applications of the couple stress model to biomechanics problems of peristaltic transport and blood flow in the microcirculation by Mekheimer [6]. Recently many authors have studied the effect of couple stress on different problems (see for example: Samuel et al. [7] and Turkyilmazoglu [8]).

There are some applications in biophysical flow through porous channels with expanding or contracting walls, such as air and blood circulation in the respiratory system, pulsating diaphragms, artificial dialysis, filtration, blood flow and binary gas diffusion. For these applications many authors studied the flow through a porous channels for different models such as analysis of some magnetohydrodynamic flows of third order fluid [9], non-Newtonian nanofluids flow through a porous medium between two coaxial cylinders with heat transfer and variable viscosity [10] and magnetohydrodynamic flow of water/ethylene glycol based nanofluids with natural convection [11]. An unsteady flows in a semi-infinite contracting or expanding pipe studied firstly by Uchida and Aoki [12]. Also, Ohki [13] investigated the unsteady flow in a porous, elastic, circular tube with contracting or expanding walls in an axial direction. A series solution to an unsteady flow in a contracting or expanding pipe is discussed by Bujurke et al. [14]. Numerical and asymptotical solutions for moderate large Reynolds numbers obtained by Majdalani and Zhou [15]. Also, Dinarvand [16] studied viscous flow with low seepage Reynolds number through slowly expanding or contracting porous walls: a model for transport of biological fluids through vessels.

No-slip condition was not have longer valid at the permeable surface, Beavers and Joseph [17] reported mass flux experiments and proved that a non-zero tangential (slip) velocity on a permeable boundary surface has effect. Some experimental and theoretical studies stated that slip condition could not be ruled out as an important element to understanding of certain characteristic flow [18]. Using a statistical approach, Saffman [19] derived the slip velocity form. Isenberg [20] posited slip condition in his study of blood flow in capillary tubes. Recently, Zhang and Jia [21] studied the first and second order Navier–Stokes equations accurate slip boundary conditions for describing the two-dimensional gaseous steady laminar flow between two plates. Ramos [22] obtained an asymptotic analytical solution for an incompressible fluid flow in channel with a slip length that depended on the pressure and/or the axial pressure gradient. Also some authors studied the effect of slip condition on some difference problems such that peristaltic flow of Jeffrey fluid model in a three dimensional rectangular duct [23], flow of non-Newtonian fluid with variable viscosity through a porous medium in an inclined channel [24] and non-Newtonian MHD fluid in porous space [25].

Lie group analysis (Lie point symmetries) method is an important method for find exact solutions of ordinary and partial differential equations by using transformations groups (similarity transformations) which introduced firstly by Sophus Lie [26]. The groups of continuous transformations that leave a given family of invariant equations are defined as the symmetries (isovector fields). The symmetry transformation is reduced the independent variables from n to $n-1$ variables [27]. Many authors have been obtained the exact solutions for some problems in fluid mechanics by using Lie group analysis method. Boutros et al. [28] studied Lie-group method solution for two-dimensional viscous flow for an expanding or contracting walls with weak permeability. Mekheimer et al. obtained the exact solutions for a couple stress fluid with heat transfer, an electrically conducting Jeffrey fluid, micro-polar fluid through a porous medium and hydro-magnetic Maxwell fluid through a porous medium [29–32], also Shahzad et al. [33] use this method to find the analytical solution of a micro-polar fluid.

The main goal of this paper is to find the analytical and approximate solutions for a magneto couple stress fluid flow in a porous channel with expanding and contracting walls using duple perturbation and Adomian methods. In Section 3, the basic roles of the Lie group analysis method are given and used to calculate the isovector field of our equations. The analytical solution (duple perturbation) corresponding to the nonlinear ordinary differential equation obtained in Section 4. Adomian decomposition method is used to obtain the solution of our ODE in Section 5. Finally, the graphs for velocity components and the pressure distribution presented for different values of the physical and geometric parameters are plotted and discussed.

2. Equations of motion

Consider an unsteady two-dimensional motion of an incompressible magneto couple stress fluid in a porous semi-infinite channel with expanding or contracting walls.

The distance $2a(t)$ between channel's walls is very small with respect to the width and length of the channel. The channel is closed from one end by a complicated solid membrane. Walls have equal permeability V_w and expand or contract uniformly at a time-dependent rate $\dot{a}(t)$, as shown in Fig. 1. We take $\widehat{x}$ and $\widehat{y}$ to be co-ordinate axes parallel and perpendicular to the channel walls and assume $\widehat{u}$ and $\widehat{v}$ to be the velocity components in the $\widehat{x}$ and $\widehat{y}$ directions respectively. The governing equations are expressed as follows,

$$\begin{cases} \dfrac{\partial \widehat{u}}{\partial \widehat{x}} + \dfrac{\partial \widehat{v}}{\partial \widehat{y}} = 0, \\ \rho\left(\dfrac{\partial \widehat{u}}{\partial t} + \widehat{u}\dfrac{\partial \widehat{u}}{\partial \widehat{x}} + \widehat{v}\dfrac{\partial \widehat{u}}{\partial \widehat{y}}\right) = -\dfrac{\partial \widehat{p}}{\partial \widehat{x}} + \mu\nabla^2\widehat{u} - \eta\nabla^4\widehat{u} - \sigma B_0^2\widehat{u}, \\ \rho\left(\dfrac{\partial \widehat{u}}{\partial t} + \widehat{v}\dfrac{\partial \widehat{v}}{\partial \widehat{x}} + \widehat{v}\dfrac{\partial \widehat{v}}{\partial \widehat{y}}\right) = -\dfrac{\partial \widehat{p}}{\partial \widehat{y}} + \mu\nabla^2\widehat{v} - \eta\nabla^4\widehat{v}, \end{cases} \tag{1}$$

where $\nabla^2 = \dfrac{\partial^2}{\partial \widehat{x}^2} + \dfrac{\partial^2}{\partial \widehat{y}^2}$, $\nabla^4 = \nabla^2(\nabla^2)$, and $\widehat{p}(\widehat{x}, \widehat{y})$ is the pressure distribution. Here ρ, μ, σ, B_0 and η are mass density, coefficient of viscosity, electrical conductivity of the fluid, magnetic field and couple-stress parameter.

The boundary conditions of our problem will be

$$\begin{aligned} &(i) \quad \frac{\partial^2 \widehat{u}}{\partial \widehat{y}^2} = 0, \quad \widehat{u} = -\frac{\sqrt{k}}{\iota}\frac{\partial \widehat{u}}{\partial \widehat{y}}, \quad \widehat{v} = -V_w = -A\,\dot{a}, \quad at \\ &\quad \widehat{y} = a(t), \\ &(ii) \quad \frac{\partial^3 \widehat{u}}{\partial \widehat{y}^3} = 0, \quad \frac{\partial \widehat{u}}{\partial \widehat{y}} = 0, \quad \widehat{v} = 0, \quad at \quad \widehat{y} = 0, \\ &(iii) \quad \widehat{u} = 0, \quad at \quad \widehat{x} = 0, \end{aligned} \tag{2}$$

where ι is a dimensionless constant which depends on the pore size of the permeable material, k is the specific permeability of the porous medium. Take the stream function $\widehat{\psi}(\widehat{x}, \widehat{y}, t)$ such