Journal of the Egyptian Mathematical Society (2016) 000, 1–6

Egyptian Mathematical Society
Journal of the Egyptian Mathematical Society
www.etms-eg.org
www.elsevier.com/locate/joems

Original Article

A novel fractional technique for the modified point kinetics equations☆

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Received 3 January 2016; revised 19 February 2016; accepted 21 February 2016
Available online xxx

Keywords

Point kinetics equations;
Multi-group of delayed neutrons;
Relaxation time effects;
Riemann–Liouville definition;
Fractional calculus

Abstract A fractional model for the modified point kinetics equations is derived and analyzed. An analytical method is used to solve the fractional model for the modified point kinetics equations. This methodical technique is based on the representation of the neutron density as a power series of the relaxation time as a small parameter. The validity of the fractional model is tested for different cases of step, ramp and sinusoidal reactivity. The results show that the fractional model for the modified point kinetics equations is the best representation of neutron density for subcritical and supercritical reactors.

2010 Mathematics Subject Classifications: 81V35; 82D75; 34A08

1. Introduction

The neutron density and the precursor concentrations of delayed neutrons at the center of a homogeneous nuclear reactor are described by a system of stiff coupled linear and/or nonlinear differential equations. An important peculiarity of the reactor kinetics is the stiffness of the system. A host of mathematical methods are developed to solve this system as a function of neutron density with different energy groups of delayed neutrons. Although, it is still currently requires more effort from the scientists to develops a new mathematical techniques and computational scheme to overcome this problem. The continuous indication of the neutron density and its rate of change are important for the safe startup, accurate determination of reactivity effects and operation of reactors. Recently, the interest of the nuclear reactor scientists is the development and analysis of different versions and approximations of the fractional-order point reactor kinetics model for a nuclear reactor "fractional neutron point kinetics equations (FNPKE)". For example: Espinosa-Paredes, Polo-Labarrios, et al. [1–7]; Ray and Patra [8]; Nowak

☆ This work was presented in the International Conference for Mathematics and Applications (ICMA15), 27 – 29 Dec. 2015, Cairo, Egypt. (DEA-11)

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Peer review under responsibility of Egyptian Mathematical Society.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.joems.2016.02.001

et al. [9,10]; and Schramm et al. [11]. Espinosa-Paredes et al. [1] is the first scientific group derived the FNPKE. Aboanber and Nahla commented on the paper [1] through letter to editor [12,13]. In this letter, the corrected form for the fractional neutron point kinetics equations is developed. In this work we focus on the derivation of the fractional modified point kinetics equations (FMPKE) and its analytical solution. The developed solution of the FMPKE is based on the representation of the neutron density as a power series in terms of the relaxation time as a small parameter, which is less than 10^{-3} (s).

The presented paper is organized as follows: Section 2 contains the derivation of the fractional modified point kinetics equations. An analytical method based on the representation of the neutron density as a power series of a small parameter is presented in Section 3. A comparison between the neutron density of the fractional modified point kinetics equations and the point kinetics equations is discussed in Section 4. The conclusion is suggested in Section 5.

2. Fractional modified point kinetics equations (FMPKE)

The explicit forms taken by the neutron conservation equation and the corresponding equation for the current density in the one-speed case are [14]

$$\frac{1}{v}\frac{\partial}{\partial t}\Phi(\mathbf{r},t)+\nabla.\mathbf{J}(\mathbf{r},t)+\Sigma_t\Phi(\mathbf{r},t)=\Sigma_s\Phi(\mathbf{r},t)+S(\mathbf{r},t), \quad (1)$$

$$\frac{1}{v}\frac{\partial}{\partial t}\mathbf{J}(\mathbf{r},t)+\nabla.\int_{4\pi}\hat{\Omega}\hat{\Omega}\varphi(\mathbf{r},\hat{\Omega},t)d\hat{\Omega}+\Sigma_t\mathbf{J}(\mathbf{r},t) =\bar{\mu}_0\Sigma_s\mathbf{J}(\mathbf{r},t)+S_1(\mathbf{r},t) \quad (2)$$

where $\varphi(\mathbf{r},\hat{\Omega},t)$ is the time dependent angular flux, as a function of space $\mathbf{r}$ and neutron direction of motion $\hat{\Omega}$, $\Phi(\mathbf{r},t)\equiv\int_{4\pi}\varphi(\mathbf{r},\hat{\Omega},t)d\hat{\Omega}$ is zero moment of the angular flux represents the neutron density, whereas the first moment of the angular flux $\mathbf{J}(\mathbf{r},t)\equiv\int_{4\pi}\hat{\Omega}\varphi(\mathbf{r},\hat{\Omega},t)d\hat{\Omega}$ represents the neutron current density, v is the neutron speed, Σ_t is the total cross section, Σ_s is the scattering cross section, $\bar{\mu}_0=\langle\hat{\Omega}.\hat{\hat{\Omega}}\rangle$ is the average scattering angle cosine, $S(\mathbf{r},t)\equiv\int_{4\pi}S(\mathbf{r},\hat{\Omega},t)d\hat{\Omega}$ is the neutron source term and $S_1(\mathbf{r},t)\equiv\int_{4\pi}\hat{\Omega}S(\mathbf{r},\hat{\Omega},t)d\hat{\Omega}$ is first moment of the neutron source.

Assume that the angular flux is only linearly anisotropic and the neutron source is isotropic, that is $\nabla.\int_{4\pi}\hat{\Omega}\hat{\Omega}\varphi(\mathbf{r},\hat{\Omega},t)d\hat{\Omega}=\frac{1}{3}\nabla\Phi(\mathbf{r},t)$ and $S_1(\mathbf{r},t)=0$. The system of Eqs. (1) and (2) take the following form

$$\frac{1}{v}\frac{\partial\Phi(\mathbf{r},t)}{\partial t}+\nabla\cdot\mathbf{J}(\mathbf{r},t)+\Sigma_a\Phi(\mathbf{r},t)=S(\mathbf{r},t) \quad (3)$$

$$\frac{1}{v}\frac{\partial}{\partial t}\mathbf{J}(\mathbf{r},t)+\Sigma_{tr}\mathbf{J}(\mathbf{r},t)+\frac{1}{3}\nabla\Phi(\mathbf{r},t)=0 \quad (4)$$

where $\Sigma_a=\Sigma_t-\Sigma_s$ is the time dependent absorption cross section, Σ_t is the total cross section, Σ_s is the scattering cross section, $\Sigma_{tr}=\Sigma_t-\bar{\mu}_0\Sigma_s$ is the transport cross section.

Eqs. (3) and (4) representing the P_1 approximation (i.e. the one-speed approximation). To simplify this system of equations, Fick's suggested for Eq. (4) that the time derivative $\frac{1}{v}\frac{\partial}{\partial t}\mathbf{J}(\mathbf{r},t)$ can be neglected in comparing with the other terms [14], which contradicts Cattaneo's law [15]. Let us divide Eq. (4) by Σ_{tr} and taking the fractional derivative on the first term as follows [16,17]:

$$\tau^\kappa\frac{\partial^\kappa}{\partial t^\kappa}\mathbf{J}(\mathbf{r},t)+\mathbf{J}(\mathbf{r},t)=-D\nabla\Phi(\mathbf{r},t) \quad (5)$$

where $D=\frac{1}{3\Sigma_{tr}}$ is the neutron diffusion coefficient and $\tau=\frac{1}{v\Sigma_{tr}}=\frac{3D}{v}$ is the relaxation time.

Substituting from Eq. (5) into Eq. (3) yields

$$\tau^\kappa\frac{\partial^\kappa}{\partial t^\kappa}\left[\frac{1}{v}\frac{\partial}{\partial t}\Phi(\mathbf{r},t)+\Sigma_a\Phi(\mathbf{r},t)-S(\mathbf{r},t)\right]+\frac{1}{v}\frac{\partial}{\partial t}\Phi(\mathbf{r},t) +\Sigma_a\Phi(\mathbf{r},t)-S(\mathbf{r},t)-D\nabla^2\Phi(\mathbf{r},t)=0 \quad (6)$$

The neutron diffusion equations with delayed neutrons are obtained by adding the source term of reactor kinetics with I groups of delayed neutrons as follow:

$$\tau^\kappa\frac{\partial^\kappa}{\partial t^\kappa}\left[\frac{1}{v}\frac{\partial}{\partial t}\Phi(\mathbf{r},t)+(\Sigma_a-(1-\beta)\nu\Sigma_f)\Phi(\mathbf{r},t) -\sum_{i=1}^{I}\lambda_iC_i(\mathbf{r},t)\right]+\frac{1}{v}\frac{\partial}{\partial t}\Phi(\mathbf{r},t)+[\Sigma_a-(1-\beta)\nu\Sigma_f]\Phi(\mathbf{r},t) -\sum_{i=1}^{I}\lambda_iC_i(\mathbf{r},t)-D\nabla^2\Phi(\mathbf{r},t)=0 \quad (7)$$

where $S(\mathbf{r},t)=(1-\beta)\nu\Sigma_f\Phi(\mathbf{r},t)+\sum_{i=1}^{I}\lambda_iC_i(\mathbf{r},t)$, $\beta=\sum_{i=1}^{I}\beta_i$ is total fraction of delayed neutrons, β_i is the fraction of i-group of delayed neutrons, ν is the mean number of fission neutrons, Σ_f is the fission cress section, λ_i is the decay constant of i-group of delayed neutrons, I is the total number of delayed neutron groups and $C_i(\mathbf{r},t)$ is the precursor concentrations of i-group of delayed neutrons which satisfy the following equations

$$\frac{\partial C_i(\mathbf{r},t)}{\partial t}=\beta_i\nu\Sigma_f\Phi(\mathbf{r},t)-\lambda_iC_i(\mathbf{r},t), \quad i=1,2,\ldots,I. \quad (8)$$

Consider that:

$$\Phi(\mathbf{r},t)=vp(t)\phi(\mathbf{r}), \quad C_i(\mathbf{r},t)=c_i(t)\phi(\mathbf{r}) \quad (9)$$

where $\phi(\mathbf{r})$ is the fundamental function, which is obtained from the diffusion equation:

$$\nabla^2\phi(\mathbf{r})+B_g^2\phi(\mathbf{r})=0 \quad (10)$$

B_g^2 is the geometric buckling appropriate for the reactor geometry (Table 3.3, page (60), [18]).

Using Eq. (10) in Eqs. (7) and (8) leads to the fractional modified point kinetics equations with multi-group delayed neutrons as

$$\tau^\kappa\frac{d^\kappa}{dt^\kappa}\left[\frac{dp(t)}{dt}-\left(\frac{\rho}{\Lambda}-\mu+\alpha\right)p(t)-\sum_{i=1}^{I}\lambda_ic_i(t)\right] +\frac{dp(t)}{dt}-\left(\frac{\rho}{\Lambda}-\mu\right)p(t)-\sum_{i=1}^{I}\lambda_ic_i(t)=0 \quad (11)$$

$$\frac{dc_i(t)}{dt}=\mu_ip(t)-\lambda_ic_i(t), \quad i=1,2,3,\ldots,I \quad (12)$$

where $p(t)$ is the neutron density, $\rho=\frac{\nu\Sigma_f-\Sigma_a(1+L^2B_g^2)}{\nu\Sigma_f}$ is the time dependent reactivity, $L^2=\frac{D}{\Sigma_a}$ is the diffusion length, $\Lambda=\frac{1}{\nu v\Sigma_f}$ is the prompt neutron generation time, $\alpha=vDB_g^2$, $\mu_i=\frac{\beta_i}{\Lambda}$ and the initial conditions of this differential equations are $p(0)=p_0$, $c_i(0)=\frac{\mu_i}{\lambda_i}p_0$, $i=1,2,3,\ldots,I$.

Please cite this article as: A.E. Aboanber, A.A. Nahla, A novel fractional technique for the modified point kinetics equations, Journal of the Egyptian Mathematical Society (2016), http://dx.doi.org/10.1016/j.joems.2016.02.001