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Noether symmetry theory of fractional order constrained Hamiltonian systems based on a fractional factor

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Abstract

In this paper, we study the Noether Symmetries and conserved quantities of fractional order constrained Hamiltonion systems based on a fractional factor. Firstly, we put forward the calculation method of fractional derivative by the fractional factor, and give the variational problem of fractional systems; Secondly, according to the regular action quantity under the infinitesimal transformation for invariance, we give the definition of Noether symmetric transformation and the criterion equation; Further, according to the relation between symmetries and conserved quantities, we obtain the Noether theorem and its inverse problem. Finally, an example is given to illustrate the application of the result. The research shows that it keeps natural height consistency in the form with the classical integer order constrained mechanical systems by using the derivative definition with fractional factor, the fractional factor can establish the connection between the fractional order systems and the integer order systems.

Keywords: Fractional factor; Fractional order constrained hamiltonian systems; Noether symmetry; Inverse problem

1. Introduction

Fractional calculus is almost simultaneous with integer calculus. It is widely used in physics, chemistry, biology, economics and so on. Many scientists and engineers believe that using fractional differential equations to describe the objective world of matter is more true [1]. In 1996, Riewe had studied the dynamic system with nonconservative forces, the fractional Euler–Lagrange equation and fractional Hamilton equation are established [2,3]. Frederico and Torres introduced the concept of fractional conservation, and gave the fractional Nother theorem based on the invariance of the fractional variational problems [4–8]. Atanacković studied the variational invariance under the definition of Riemann-Liouville fractional derivative, the invariance condition and Noether theorem of the system are presented [9]. In 2005, EI-Nabulsi proposed a new modeling method:class fractional variational method, only one parameterise introduced, the resulting that Euler–Lagrange equation is simple and similar to the classical equation in form [10,11]. On the basis of EI-Nabulsi's research, Zhang yi studied the fractional variational problem more deeply, and the previous results have been further extended [12,13]. Khalil and Abdeljawad proposed a new definition and basic properties of fractional calculus, which is

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consistent with the definition of integer order derivatives, it was called conformable fractional derivatives [14,15]. Fu jingli studied the Noether and Lie symmetries and conserved quantities of fractional order Lagrange and Hamiltonian systems based on joint Caputo derivatives and conformable fractional derivatives [16,17]. Fu jingli studied the Routh equation and the cyclic integral problem of the fractional order Lagrange system based on a fractional factor derivatives, it obtains some new results [18], these results are similar with integer order that it can not easy to obtain in the previous fractional order dynamics. The other research literature about the recent fractional order dynamics systems have [19–21].

Study on the symmetries of fractional nonsingular systems have been obtained some progress. However, under the Legendre transformation, when the singular Lagrange system transits to the phase space and is described by the Hamiltonian systems, there exists an inherent constraint between its canonical variables, which is called the constrained Hamiltonian systems [22]. Many important dynamical systems in reality are the model of constrained Hamiltonian systems [23], such as supersymmetry, supergravity, electromagnetic field, relativistic motion of the particle, superstring and Yang-Mills field etc. However, the research on the variational problem and the symmetries of fractional order constraint Hamiltonian systems is rarely reported, meanwhile, the familiar fractional derivative calculation is complex and involves the operation of special function. It is well known that the Noether theorem plays an important role in classical mechanics, especially in modern science and technology. In this paper, a new definition of fractional derivative by a fractional factor is given, and the Noether symmetry theorem of fractional singular system is further established in phase space. It provides the integral theory of fractional singular systems and verification method for various numerical algorithms

2. Fractional factor and fractional derivative

As everyone knows, Riemann-Liouville fractional derivative, Grunwald-Letnikov fractional derivative and Caputo fractional derivative are the integral form of the definition, it has only linear optimality, but its basic properties of calculus with integer order calculus is not a natural consistency. Recently, a novel fractional derivative whose definition and important properties follows [18].

The α order derivative $(0<\alpha<1)$ of function $y=f(t)$, which is defined with fractional factor:

$$D_\alpha(f)=f^\alpha(t)=\lim_{\varDelta_\alpha t\to 0}\frac{f\left(t+e^{-(1-\alpha)t}\varDelta_\alpha t\right)-f(t)}{\varDelta_\alpha t}=\frac{df(t)}{d_\alpha t}$$
$$\varDelta t=e^{-(1-\alpha)t}\varDelta_\alpha t$$
$$dt=e^{-(1-\alpha)t}d_\alpha t \tag{1}$$

Fractional integral based on fractional factor can be used as:

$$I_\alpha^{ab}(f)=\lim_{\max\{\varDelta t_i\}\to 0}\sum_{i=1}^{n}f(\xi_i)\varDelta_\alpha t=\int_\alpha^{ab}f(t)d_\alpha t$$
$$=\int_a^b e^{(1-\alpha)t}f(t)dt \tag{2}$$

The exchange relations between isochronous variational and fractional order operators, and the fractional differential rule of composite functions are:

$$\delta D_\alpha q=\delta\left(e^{-(1-\alpha)t}\frac{dq}{dt}\right)=D_\alpha\delta q=e^{-(1-\alpha)t}\frac{d\delta q}{dt}$$
$$D_\alpha(fg)=D_\alpha(f)\cdot g+D_\alpha(g)\cdot f \tag{3}$$

3. Variational problems and equations of motion

The form of fractional order mechanical systems is determine by generalized coordinates $q_s(s=1,2,...,n)$, the Lagrange function of the system is $L(t,\mathbf{q},D_\alpha\mathbf{q})$ $0<\alpha<1$, introduced the generalized momentum $p_{\alpha s}$ and fractional order Hamilton function H^α are:

$$p_{\alpha s}=\frac{\partial L}{\partial D_\alpha q_s}$$
$$H^\alpha(t,\mathbf{q},\mathbf{p}_\alpha)=p_{\alpha s}D_\alpha q_s-L(t,\mathbf{q},D_\alpha\mathbf{q}) \tag{4}$$

If the rank of hessian matrix of L is $r<n$, that is $\det(h_{sk})=\left|\left[\frac{\partial^2 L}{\partial D_\alpha q_s\partial D_\alpha q_k}\right]\right|=0$, so the systems are a fractional singular systems. When the Lagrange describes transition to the Hamiltonian systems, all $D_\alpha q_s(t)$ can not be solved by Formula (4) as the function of $t,\mathbf{q},\mathbf{p}_\alpha$, an inherent constraint exists between regular variables ($q_s,p_{\alpha s}$ is not complete independence):

$$\varphi_j(t,\mathbf{q},\mathbf{p}_\alpha)=0(j=1,2,...,n-r) \tag{5}$$

The variational problem of the fractional order Hamiltonian systems can be described as: