ARTICLE IN PRESS

+ MODEL

Available online at www.sciencedirect.com

ScienceDirect

Karbala International Journal of Modern Science xx (2017) 1−11

http://www.journals.elsevier.com/karbala-international-journal-of-modern-science/

Survey on density of states and saturation effect of spectrum for an energy-dependent harmonic interaction

Abdelmalek Boumali [a,*], Samia Dilmi [b], Soroush Zare [c], Hassan Hassanabadi [d]

[a] *Laboratoire de Physique Appliquée et Théorique, University Larbi Tébessi, Tébessa, 12000, W. Tébessa, Algeria*

[b] *Faculté des Sciences Exactes, Université Hama Lakhdar D'El-Oued, B.P. 789, El-Oued, 39000, Algeria*

[c] *Department of Basic Sciences, Islamic Azad University, North Tehran Branch, Tehran, Iran*

[d] *Physics Department, Shahrood University of Technology, Shahrood, Iran*

Received 4 May 2017; revised 7 September 2017; accepted 7 September 2017

Abstract

In this article, we test the influence of the modification of the scalar product, found in the problems of the energy-dependent potential, on the physical properties of the harmonic oscillator in one dimension. For this, we first discuss the effect of this change on the thermodynamic properties of this oscillator, and then investigate the parameters of Fisher and Shannon of quantum information. For the second problem, the dependence of these parameters with the density of state, is the main reason of our choice to study the influence of this type of potential on these properties. Finally, the well-know uncertainty relation of Cramer−Rao is well recovered.

Keywords: Fisher parameter; Shannon entropy; Partition function; Cramer−Rao uncertainty relation

1. Introduction

Wave equations with energy dependent potentials have been view for long time. They can be seen in Klein−Gordon equation considering particle in an external electromagnetic field [1]. Arising from momentum dependent interactions, they also appeared in non-relativistic quantum mechanics, as shown by Green [2] for instance Pauli−Schrödinger equation possesses another example [3,4]. Sazdjian [5] and Formanek et al. [6] have noted that the density probability, or the scalar product, has to be modified with respect to the usual definition, in order to have a conserved norm. Garcia-Martinez et al. and Lombard made an investigation on Schrödinger equation with energy-dependent potentials by solving them exactly in one and three dimensions [7,8]. Hassanabadi et al. studied the D-dimensional Schrödinger equation for an energy-dependent Hamiltonian that linearly depends on energy and quadratic on the relative distance [9]. They also studied the Dirac equation for an energy-dependent potential in the presence of spin and pseudospin symmetries with arbitrary spin−orbit quantum number. We calculated the corresponding

* Corresponding author.

E-mail addresses: boumali.abdelmalek@gmail.com (A. Boumali), dilmisamia@gmail.com (S. Dilmi), soroushzrg@gmail.com (S. Zare), h.hasanabadi@shahroodut.ac.ir (H. Hassanabadi).

Peer review under responsibility of University of Kerbala.

http://dx.doi.org/10.1016/j.kijoms.2017.09.001

eigenfunctions and eigenvalues of a non-relativistic energy-dependent system [10] was studied by Lombard and Mareš [11]. They considered systems of N bosons bounded by two-body harmonic interactions, whose frequency depends on the total energy of the system. Other interesting related works can be found in Refs. [12−15] and references therein.

Presence of the energy dependent potential in a wave equation has several non-trivial implications. The most obvious one is the modification of the scalar product, necessary to ensure the conservation of the norm. This modification modified some behavior or physical properties of a physical system: here we note that this question, in best of our knowledge, has not been considered in the literature. In this context, the main goal of this paper is studying of the effects of the modified scalar product arising in the energy-dependent harmonic potential problem. For this, we are focused on the study of: (i) the thermal properties of the 1D harmonic oscillator and, (ii) the Fisher and Shannon parameters of quantum information and the corresponding solutions are obtained in Sec. 2, the harmonic oscillator with an energy-dependent frequency has been considered and the corresponding solutions are obtained. Then the effect of the modified scalar product on the thermal properties are discussed in Sec. 3. In Sec. 4, we study the effects of energy dependence on the Fisher and Shannon parameters of quantum information. Sec. 5 will be a conclusion.

2. One-dimensional harmonic oscillator with an energy-dependent potential

2.1. The eigenfunctions and eigenvalues

We consider the harmonic oscillator potential with an energy-dependent frequency

$$H=\frac{p^2}{2}+\frac{1+\gamma E^{\nu}}{2}x^2, \tag{1}$$

where γ is a parameter (not necessarily small, though most of our investigations are dedicated to small γ) and $m=\hbar=1$. Consequently, the time-independent Schrödinger equation for the oscillator can be written as

$$\left\{-\frac{1}{2}\frac{d^2}{dx^2}+\frac{(1+\gamma E^{\nu})}{2}x^2\right\}\psi(x)=E\psi(x). \tag{2}$$

The eigensolutions of such a system are

$$\psi_n(x)=C_n e^{-\frac{\sqrt{1+\gamma E^{\nu}}x^2}{2}}H_n\left(\sqrt{\sqrt{1+\gamma E^{\nu}}}\,x\right). \tag{3}$$

with the normalization constant C_n. Now, Let us consider the following two particular cases [6]:

- First case: $\nu=1$, the eigenvalues are

$$E_{1n}=\frac{\left(n+\frac{1}{2}\right)}{2}\left[\gamma\left(n+\frac{1}{2}\right)\pm\sqrt{\gamma^2\left(n+\frac{1}{2}\right)^2+4}\right]. \tag{4}$$

- Second case: $\nu=2$, we obtain

$$E_{2n}=\pm\left(n+\frac{1}{2}\right)\sqrt{\frac{1}{1-\gamma\left(n+\frac{1}{2}\right)^2}}. \tag{5}$$

The corresponding wave function in both cases can be written in compact form as

$$\psi_{\nu n}(x)=C_{\nu n}e^{-\frac{\lambda_{\nu}x^2}{2}}H_n\left(\sqrt{\lambda_{\nu}}\,x\right), \tag{6}$$

with

$$\lambda_{\nu}=\sqrt{1+\gamma E_n^{\nu}},\qquad \nu=(1,2). \tag{7}$$

In what follows, only the positive energies are retained.

Now we are in the main goal of our study, i.e. the influence of the modified scalar product on the wave function and the corresponding energy of the energy-dependent potential, especially on the properties of eigenvalues. According to Refs. [5,6,8], the definition of the density has to be modified in order to ensure the validity of the continuity equation. For the non-relativistic Schrödinger equation, the new definition for the density of a state n is given by

$$\rho_n(x)=|\psi_n(x)|^2\left(1-\frac{dV}{dE}\right). \tag{8}$$

In order to represent a physical system, the density has to be positive definite, which means here

$$1-\frac{dV}{dE}\geq 0. \tag{9}$$

This imposes constraints on the energy dependence for the theory to be coherent: by this, we mean a theory that has the following properties: (i) the necessary modification of the definition of probability density,

Please cite this article in press as: A. Boumali et al., Survey on density of states and saturation effect of spectrum for an energy-dependent harmonic interaction, Karbala International Journal of Modern Science (2017), http://dx.doi.org/10.1016/j.kijoms.2017.09.001